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(Pseudo-)Goldstone boson interaction in $D = 2 + 1$ systems with a spontaneously broken internal rotation symmetry

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Abstract

The low-temperature properties of systems characterized by a spontaneously broken internal rotation symmetry, $O(N) \to O(N-1)$, are governed by Goldstone bosons and can be derived systematically within effective Lagrangian field theory. In the present study we consider systems living in two spatial dimensions, and evaluate their partition function at low temperatures and weak external fields up to three-loop order. Although our results are valid for any such system, here we use magnetic terminology, i.e., we refer to quantum spin systems. We discuss the sign of the (pseudo-)Goldstone boson interaction in the pressure, staggered magnetization, and susceptibility as a function of an external staggered field for general N. As it turns out, the $d = 2+1$ quantum XY model ($N = 2$) and the $d = 2+1$ Heisenberg antiferromagnet ($N = 3$), are rather special, as they represent the only cases where the spin-wave interaction in the pressure is repulsive in the whole parameter regime where the effective expansion applies. Remarkably, the $d = 2 + 1$ XY model is the only system where the interaction contribution in the staggered magnetization (susceptibility) tends to positive (negative) values at low temperatures and weak external field.

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1. Introduction

The present study is devoted to (pseudo-)Lorentz-invariant systems that are defined in two spatial dimensions and – at $T = 0$ – are characterized by a spontaneously broken rotation symmetry $O(N) \to O(N-1)$. We focus on the low-energy dynamics which is dominated by the corresponding $N-1$ Goldstone bosons. The most prominent physical realizations are the $d = 2+1$ quantum XY model ($N = 2$) and the $d = 2+1$ Heisenberg antiferromagnet ($N = 3$). In either case the relevant low-energy degrees of freedom are the spin waves or magnons.

Although there exists a vast number of publications – in particular for $N = \{2, 3\}$ – the impact of the Goldstone boson interaction onto the low-temperature behavior of these systems has been largely neglected. While the specific cases $N = \{2, 3\}$ have been addressed within effective Lagrangian field theory in Refs. [1–8], a systematic study of the nature of the interaction in systems with general N, seems to be lacking. It is the aim of the present investigation to rigorously analyze the effect of the Goldstone boson interaction in the pressure, in the staggered magnetization, and in the susceptibility, using the systematic and model-independent effective Lagrangian technique.

A method widely used to analyze the low-temperature behavior of spin systems with a spontaneously broken internal rotation symmetry is spin-wave theory, or its modified versions adapted to two spatial dimensions [9–12]. With respect to the $d = 2+1$ quantum XY model, the interested reader may also consult Refs. [13–16]. The finite-temperature properties of the $d = 2+1$ Heisenberg antiferromagnet and the $d = 2+1$ quantum XY model have furthermore been analyzed with alternative methods that include Schwinger-boson mean-field theory, Green's functions, renormalization group methods, and Monte-Carlo simulations. Concerning the $d = 2+1$ quantum XY model we refer to Refs. [17–27], while for the $d = 2+1$ Heisenberg antiferromagnet we point to Refs. [28–41]. For the general case, i.e., for arbitrary N, see Refs. [42–48].

Here we do not use spin-wave theory or any other microscopic method. Rather, our approach relies on the effective Lagrangian technique which systematically captures the low-energy physics of the system that is governed by the Goldstone bosons. Low energy – or low temperature – means that we are interested in the properties of the system below the intrinsic scale defined by the underlying microscopic model. In connection with the Heisenberg antiferromagnet or the quantum XY model, this scale is given by the exchange coupling J. Note that the spontaneously broken rotation symmetry $O(N)$ is only approximate, since we incorporate a weak external field into the low-energy description. As we discuss below, this field cannot be completely switched off in two spatial dimensions in the effective approach we pursue. Since the spontaneously broken rotation symmetry is only approximate, the Goldstone bosons are not strictly massless, but pick up a small mass or energy gap – and are hence referred to as (pseudo-)Goldstone bosons. In what follows, however, we prefer to call them Goldstone bosons, keeping in mind that they are not exactly massless.

The $d = 2+1$ quantum XY model ($N = 2$) and the $d = 2+1$ Heisenberg antiferromagnet ($N = 3$) turn out to be rather peculiar cases: they represent the only systems where the spin-wave-interaction contribution in the pressure is repulsive at low temperatures in the entire parameter regime where our effective analysis applies. Furthermore, considering the impact of the spin-wave interaction in the staggered magnetization and susceptibility, the low-temperature behavior of the $d = 2+1$ quantum XY model is quite different from any other $d = 2+1$ (pseudo-)Lorentz-invariant system with a spontaneously broken internal rotation symmetry $O(N)$: it is the only system where the interaction contribution in the staggered magnetization (susceptibility) tends to positive (negative) values at low temperatures and weak external field. Interestingly, for systems