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Monodromic vs geodesic computation of Virasoro classical conformal blocks

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Abstract

We compute 5-point classical conformal blocks with two heavy, two light, and one superlight operator using the monodromy approach up to third order in the superlight expansion. By virtue of the AdS/CFT correspondence we show the equivalence of the resulting expressions to those obtained in the bulk computation for the corresponding geodesic configuration.

1. Introduction

Conformal blocks represent important elements of any conformal field theory [1]. In general, they are certain functions of conformal dimensions $\{\Delta_i\}$ and Virasoro central charge c which are completely defined by the conformal symmetry. In the limit where c goes to infinity they are approximated by the so-called classical conformal blocks. Recently, a remarkable interpretation of the classical conformal blocks in the context of the AdS_3/CFT_2 correspondence has been inves-

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tigated [2–8]. It was shown that some class of classical CFT_2 conformal block can be described by means of a particular classical mechanics in AdS_3.

There exist different types of the classical conformal blocks depending on the behavior of the conformal dimensions as $c \to \infty$ [9]. One can distinguish between two limiting cases: conformal blocks with only heavy and only light operators. In this note we focus on the case of two heavy operators of equal conformal dimensions producing in the bulk either a conical defect or BTZ black hole [2,3,5,6]. The bulk geodesic configuration corresponding to the n-point block with only two heavy fields consists of $2n-5$ massive scalar particles propagating in the background geometry produced by the heavy operators [7]. It was pointed out in [10,8] that these configurations are nothing but the ordinary Witten exchange diagrams with the difference that there is no integration over positions of the vertices of the geodesic graph.

While the 4-point conformal block in the heavy-light approximation[1] was explicitly computed [5,6] an exact consideration of the n-point case is hampered by technical difficulties related to solving associated higher order algebraic equations. To analyze multi-particle configurations we proposed to use an additional approximation procedure with respect to a small parameter which can be chosen to be a mass of one of the particles whose worldline ends on the boundary (*i.e.*, it is a conformal dimension of one of external fields) [7]. This allows to iteratively reconstruct n-point heavy-light classical conformal block starting from the $(n-1)$-point one. Such a *super-light* approximation applies when there is a known expression for the n-point heavy-light conformal block. This is the case with the 5-point heavy-light block considered as a deformation of the exactly known heavy-light 4-point block with respect to a small classical conformal dimension of the third light operator. In this paper we use this procedure to analyze the bulk/boundary correspondence of the conformal block/geodesic Witten diagram computation of the 5-point heavy-light block in the sub-leading orders of the super-light expansion.

Our computation of the 5-point classical conformal block on the boundary relies on the study of the monodromy properties of the auxiliary Fuchsian differential equation. It is reduced to the computation of the so-called accessory parameters which are partial derivatives of the conformal block function. In the bulk we use the geodesic approach involving a different but related set of quantities – angular momenta of external and intermediate geodesic segments, and the mechanical action of the geodesic configuration or, equivalently, the geodesic length. Analogously to the accessory parameters, the external angular momenta are defined as derivatives of the mechanical action which in turn is related to the classical conformal block [6]. We expect that the geodesic description can be successively derived from the monodromy approach by finding counterparts of the monodromy approach constituents on the bulk side. In this paper we compare the systems of algebraic equations describing both the accessory and angular parameters and find out that they are generally different but have a common physically relevant root which leads to the conformal block function.

The paper is organized as follows. In Section 2 we apply the monodromy approach to find equations on the accessory parameters for the classical 5-point conformal block. In Section 3 we formulate the super-light approximation procedure and compute the accessory parameters up to the third order in the dimension of the super-light operator. In Section 4 we perform the corresponding bulk computation. In Section 5 we discuss explicit relations between the classical

[1] For the general 4-point block the basic computation tool is Zamolodchikov recursion relations [11,12], for the recent development see [13]. Alternative recursion in the important case of the vacuum conformal blocks was proposed recently in [14]. For multi-point blocks the AGT combinatorial representation is applicable (at finite value of the central charge in rational and non-rational CFTs with Virasoro symmetry see [15–17].