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Algebraic structures on parallel M2 branes

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Abstract

In the course of closing supersymmetry on parallel M2 branes up to a gauge transformation, following the suggestion in [J. Bagger, N. Lambert, Phys. Rev. D 75 (2007) 045020, hep-th/0611108] of incorporating a gauge field which only has topological degrees of freedom, we are led to assume a certain algebraic structure for the low energy theory supposedly living on parallel M2 branes.

1. Introduction

The low-energy theory living on a single M2 brane was derived in [1,2]. This theory can also be derived from the Yang–Mills action living on a single D2 brane by dualizing one of the scalar fields [3].

In this article we investigate the 'non-Abelian' generalization of this M2 brane theory, much inspired by the work of Bagger and Lambert [4]. More specifically, we ask what requirements come from supersymmetry. Our starting point is to assume that supersymmetry can be realized on some kind of 'fields' with the usual Lorentz index structure. We do not need to know much about the internal structure to be able to analyze the supersymmetry transformations. All we in essence need, is usual gamma matrix algebra.

We put non-Abelian in quotation marks because the fields will not take values in the adjoint[1] representation of a non-Abelian Lie algebra. We also put the word 'fields' in quotation marks because it is not clear that these would be just ordinary fields.

We thus assume there are some kind of non-commuting 'fields' that take values in some algebra, and that there are certain ways of multiplying together such fields to get a new element

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[1] Since the gauge field would be in the adjoint, all other fields would also be in the adjoint due to supersymmetry.

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in the algebra. We also require that these fields are such that they reduce to the ordinary eight scalar fields plus their supersymmetric partners in the Abelian case.

Dimensional analysis suggests there is in the supersymmetry variations, products of two as well as of three fields. It is of course desirable that all products of our fields are such that they close on some internal algebra. The way we do that is by making the minimal assumption that there is a multiplication of two fields which belong to some set of fields, that we denote as $\mathcal{A}$, such that the product is in some set $\mathcal{B}$ that need not be the same as $\mathcal{A}$, though a product of three elements in $\mathcal{A}$ must yield back an element in $\mathcal{A}$. We introduce three different kinds of multiplications to be used according to whether the elements being multiplied belong to $\mathcal{A}$ or $\mathcal{B}$. Then we see what requirements closure of the supersymmetry transformations impose on these various multiplications.

2. The Abelian cases

The Abelian super-Yang–Mills SUSY transformations can be derived by dimensionally reducing $(1+9)$-dimensional super-Yang–Mills to $1+2$ dimensions. The ten-dimensional spinor is Majorana–Weyl,

$$\Gamma^{(10)}\chi = \chi. \tag{1}$$

Since we wish to prepare the ground for an up-lift to M-theory, it is desirable to look for an embedding of $SO(1,9)$ into $SO(1,10)$ in which $\Gamma^{(10)}$ is the eleventh gamma matrix. We denote the gamma matrices as Γ^M $(M = 0,\ldots,9,10)$. In M-theory, the spinor can obviously just be Majorana. But the presence of an M2 brane breaks the Lorentz symmetry as $SO(1,10) \to SO(1,2) \times SO(8)$, and we can have a Weyl spinor of $SO(8)$. Let us denote by

$$\Gamma = \Gamma^{3\ldots 9(10)} \tag{2}$$

the chirality matrix of $SO(8)$. Half the supersymmetry is broken by the M2 brane. Let us choose a convention where the unbroken supersymmetry parameters are Weyl

$$\Gamma\epsilon = \epsilon. \tag{3}$$

The broken supersymmetries gets manifested as goldstinos,

$$\Gamma\psi = -\psi, \tag{4}$$

living on the world-volume of the M2 brane. The broken translations become eight goldstone scalar fields, ϕ^A on the M2 brane. The unbroken supersymmetries relate the bosonic and fermions degrees of freedom. The Yang–Mills supersymmetry transformations, written in terms of such spinors, is given by [4]

$$\begin{aligned}
\delta\phi^a &= i\bar{\epsilon}\Gamma^a\psi,\\
\delta F_{\mu\nu} &= -2i\bar{\epsilon}\Gamma_{[\mu}\Gamma^{(10)}\partial_{\nu]}\psi,\\
\delta\psi &= \frac{1}{2}F_{\mu\nu}\Gamma^{\mu\nu}\Gamma^{(10)}\epsilon + \partial_\mu\phi^a\Gamma^\mu\Gamma_a\epsilon.
\end{aligned} \tag{5}$$