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Emergent gravity and noncommutative branes from Yang–Mills matrix models

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Abstract

The framework of emergent gravity arising from Yang–Mills matrix models is developed further, for general noncommutative branes embedded in $\mathbb{R}^D$. The effective metric on the brane turns out to have a universal form reminiscent of the open string metric, depending on the dynamical Poisson structure and the embedding metric in $\mathbb{R}^D$. A covariant form of the tree-level equations of motion is derived, and the Newtonian limit is discussed. This points to the necessity of branes in higher dimensions. The quantization is discussed qualitatively, which singles out the IKKT model as a prime candidate for a quantum theory of gravity coupled to matter. The Planck scale is then identified with the scale of $N = 4$ SUSY breaking. A mechanism for avoiding the cosmological constant problem is exhibited.

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1. Introduction

The notion of space–time which underlies the presently accepted models of fundamental matter and interactions goes back to Einstein. Space–time is modeled by a 4-dimensional manifold, whose geometry is determined by a metric with Lorentzian signature. This notion escaped the quantum revolution essentially unchanged, even though Quantum Mechanics combined with General Relativity strongly suggests a "foam-like" or quantum structure at the Planck scale.

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While some kind of quantum structure of space–time indeed arises e.g. in string theory or loop quantum gravity, a satisfactory understanding is still missing.

A different approach to this problem has been pursued in recent years, starting with some explicit quantization of space–time and attempting to construct physical models on such a background. The classical space–time $\mathbb{R}^4$ is replaced by a quantized or "noncommutative" (NC) space, where the coordinate functions x^μ satisfy nontrivial commutation relations such as $[x^\mu, x^\nu] = i\theta^{\mu\nu}$. This leads to noncommutative field theory, see e.g. [1–3]. At the semi-classical level, these commutation relations determine a Poisson structure $\theta^{\mu\nu}$ on space–time, which is fixed by construction. However, since quantized spaces are expected to arise from quantum gravity, it seems more appropriate to consider a *dynamical* Poisson structure at the semi-classical level. A straightforward generalization of General Relativity is then inappropriate; indeed any quantum structure of space–time rules out classical intuitive principles. Rather, one should look for simple models of dynamical noncommutative (or Poisson) spaces, with the hope that they will effectively incorporate gravity.

Such models are indeed available and known as matrix models of Yang–Mills type. They have the form $S = \mathrm{Tr}[X^a, X^b][X^{a'}, X^{b'}]\delta_{aa'}\delta_{bb'} + \cdots$, where indices run from 1 to D. It is well known that these models admit noncommutative spaces ("NC branes") as solutions, such as the Moyal–Weyl quantum plane $\mathbb{R}^4_\theta$; see e.g. [13–19]. However, most of the work up to now is focused on special NC branes with a high degree of symmetry. For generic NC spaces with non-constant $\theta^{\mu\nu}(x)$, it was shown in [4] that the kinetic term for any "field" coupled to the $D = 4$ matrix model is governed by an effective metric $\tilde{G}^{ab}(x) = \rho\theta^{aa'}(x)\theta^{bb'}(x)\delta_{a'b'}$, including non-Abelian gauge fields. This nicely explains the observed relation in [5] between NC $U(1)$ gauge fields and gravitational degrees of freedom, see also [6–8] for related work. Since this effective metric is dynamical, these YM Matrix Models contain effectively some version of gravity, thus realizing the idea that gravity should emerge from NC gauge theory [5,9]. As argued in [4], an effective action for gravity is induced upon quantization, with the remarkable feature that the "would-be cosmological term" decouples from the model due to the constrained class of metrics. This makes the mechanism of induced gravity feasible at the quantum level, and suggests that the Newton constant respectively the Planck scale is related to an effective UV-cutoff of the model. A detailed analysis taking into account UV/IR mixing [10] and fermions [11] singles out the $N = 4$ supersymmetric extensions of the model, where such a cutoff is given by the scale of $N = 4$ SUSY breaking. This amounts to $D = 10$, which is nothing but the IKKT model [12], originally proposed as a nonperturbative definition of IIB string theory.[1]

In the present paper, we develop the framework for emergent gravity on general NC branes with nontrivial embedding in $\mathbb{R}^D$. This works out very naturally, leading to a simple generalization of the effective metric which is strongly reminiscent of the open string metric [27], involving the general Poisson tensor and the embedding metric. We establish in Section 2 the relevant geometry, find the semi-classical form of the bare matrix model action for general NC branes in $\mathbb{R}^D$, and obtain covariant equations of motion. This generalizes the well-known case of flat or highly symmetric branes to the generic case, and shows how the would-be $U(1)$ gauge field is absorbed in the effective metric on the brane. In Section 3, the Newtonian limit of emergent gravity is studied in detail. It turns out that even though it is possible to reproduce the Newtonian potential

[1] As such, the presence of gravity in this model is expected and to some extent verified, cf. [12,13,20–22,24,25]. However, what is usually considered are effects of $D = 10$ (super)gravity, modeled by interactions of separated "D-objects", represented by block-matrices. In contrast, emergent NC gravity describes interactions within (generic) NC branes in this model. Evidence for gravity on simple NC branes was obtained previously in [24,26].