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Super-renormalizable and finite gravitational theories

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Abstract

We hereby introduce and extensively study a class of non-polynomial higher derivative theories of gravity that realize a ultraviolet (UV) completion of Einstein general relativity. These theories are unitary (ghost free) and at most only one-loop divergences survive. The outcome is a class of theories super-renormalizable in even dimension and finite in odd dimension. Moreover, we explicitly prove in $D = 4$ that there exists an extension of the theory that is completely finite and all the beta functions vanish even at one-loop. These results can be easily extended in extra dimensions and it is likely that the higher dimensional theory can be made finite, too. Therefore we have the possibility for "finite quantum gravity" in any dimension.

1. Introduction

Quantum abelian and non-abelian gauge theories as the most complete embodiment of particle physics are all compatible with two guiding principles: "renormalizability" and "perturbative theory" in the quantum field theory framework. This is the achievement of a consistent quantum field theory for all but one fundamental interactions. Indeed, gravity seems to elude so far these patterns and many authors suggested ingenious solutions to one of the biggest puzzles of our days, but none is completely satisfactory. The major obstacle, when we try to interface gravity and quantum mechanics is that Einstein's dynamics is "non-renormalizable", but in principle

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there is nothing inconsistent between them. Just like for Fermi theory of weak interactions, quantum Einstein's gravity is solid and calculable in the effective field theory framework. The cutoff scale is naturally given for it by Planck energy. On the other hand, when the theory is made renormalizable by adding higher derivative operators, it is no more unitary and shows up propagation of ghost states. In the end there is a strong tension between renormalizability and unitarity in gravitational theories. The key ingredient to overcome this problem is to introduce a non-polynomial (or non-local) "kinetic" extension of Einstein's gravity. We here use the terminology "kinetic part" for operators linear or quadratic in the gravitational curvature, and "potential" for a finite sum of all other local operators in the action.

It is clear from the discussion above that we regard as crucial to find a "new theory of gravity", which is unitary and renormalizable or even finite at quantum level. Moreover we require that such theory is free of singularities at the classical level. We indeed believe in a one to one correspondence between singularities in classical theory and quantum divergences.

The aim of this work is to extend *classical* Einstein–Hilbert theory to make gravity compatible with the above guiding principles (renormalizability and perturbative theory) in the "quantum field theory framework". We start with a new unitary non-polynomial higher derivative theory for gravity in a multidimensional spacetime [1–14] (see also [15–25]). Next we show that it is possible to restrict to a subclass of theories, in which at quantum level only one-loop divergences survive. Moreover, in such theories these one-loop divergences can be removed by introducing, for example in $D = 4$, extra operators that are cubic or quartic in the curvature, typically of the form $O(\mathcal{R}^2\Box^{\gamma-1}\mathcal{R})$, $O(\mathcal{R}^2\Box^{\gamma-2}\mathcal{R}^2)$. We end up with a completely finite theory of quantum gravity, because all the beta functions can be consistently made to vanish by choosing proper coefficients for specially added operators. The result can be extended in any dimension and for a more complicated curvature potential. In this paper we systematically complete the previous work on polynomial [36,37] and non-polynomial super-renormalizable quantum gravity [1–12,14]. Our work is also inspired by numerous works on nonlocal infrared modifications of gravity [27–35].

Definitions — The metric tensor $g_{\mu\nu}$ has signature $(- + \ldots +)$ and the curvature tensors are defined as follows: $R^{\mu}_{\nu\rho\sigma} = -\partial_\sigma \Gamma^{\mu}_{\nu\rho} + \ldots$, $R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}$, $R = g^{\mu\nu}R_{\mu\nu}$. With symbol $\mathcal{R}$ we generally denote one of the above curvature tensors.

2. Modern gravity

In this section we introduce a "New Gravity" theory in a D-dimensional spacetime assuming the following consistency requirements:

1. Unitarity. A general theory is well defined, if the corresponding propagator has only first poles with real masses (no tachyons) and with positive residues (no ghosts).
2. Super-renormalizability or Finiteness. This hypothesis makes consistent the theory at quantum level on the same footing as for all the other fundamental interactions.
3. Lorentz invariance. This is a symmetry of nature well tested experimentally beyond the Planck mass.
4. The classical energy conditions can only be violated, because higher-derivative operators are present in the classical theory. This property is crucial to avoid singularities, that plague almost all the solutions of Einstein's gravity [15,41–51].