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LQG vertex with finite Immirzi parameter

Jonathan Engle [a], Etera Livine [b,*], Roberto Pereira [a], Carlo Rovelli [a]

[a] *Centre de Physique Théorique de Luminy, Case 907, F-13288 Marseille, France*
[b] *Laboratoire de Physique, ENS Lyon, CNRS UMR 5672, 46 Allée d'Italie, 69364 Lyon, France*

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Abstract

We extend the definition of the "flipped" loop-quantum-gravity vertex to the case of a finite Immirzi parameter γ. We cover both the Euclidean and Lorentzian cases. We show that the resulting dynamics is defined on a Hilbert space isomorphic to the one of loop quantum gravity, and that the area operator has the same discrete spectrum as in loop quantum gravity. This includes the correct dependence on γ, and, remarkably, holds in the Lorentzian case as well. The *ad hoc* flip of the symplectic structure that was required to derive the flipped vertex is not anymore required for finite γ. These results establish a bridge between canonical loop quantum gravity and the spinfoam formalism in four dimensions.

1. Introduction

The Barrett–Crane (BC) vertex, which provides a tentative definition of the quantum-gravity dynamics, has been extensively investigated during the past years [1]; its amplitude is essentially given by a Wigner $10j$ symbol. A different vertex has been recently introduced in [2,3]; its amplitude essentially given by the square of an $SU(2)$ Wigner $15j$ symbol. There are indications that this new vertex could ameliorate the properties of the BC model. First, it appears to correct an over-imposition of the constraints that was remarked in the derivation of the BC vertex. Second, it does not appear to freeze the angular degrees of freedom of the gravitational field (that is, $g_{ab}(x)$ for $a \neq b$) as it has been argued the BC model might do [4]. Third, preliminary numerical investigations appear to be consistent with the expectation that geometry wave packets

* Corresponding author.
E-mail address: erlivine@gmail.com (E. Livine).

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are propagated by this new vertex in a way consistent with Euclidean general relativity (GR) [5]. And finally, its kinematics matches exactly the one of the canonical quantization of GR, as given by loop quantum gravity (LQG) [6]. The vertex was defined in [2,3] only for the Euclidean case, and in the absence of an Immirzi parameter γ.

A key step to extend the definition of this new vertex was taken in [7], where a Lorentzian version of the vertex amplitude is constructed, still without γ. Here we extend the construction of the vertex to the general case of finite γ, both for the Euclidean and the Lorentzian sectors.

As long emphasized by Sergei Alexandrov [8], the key technical problem is how to impose the second class quantum constraints in a covariant way (see [9]). These constraints are solved in [2,3] using a master-constraint-like [10] technique. In [11], it was shown that these constraints can equivalently be solved using a different technique, based on coherent states, yielding the same result. This derivation reinforces the credibility of the approach, and opens a direct connection to the semiclassical limit.

In the same paper [11], on the other hand, it was also pointed out that considering a different class of coherent states leads to a variant of the model. This variant has been extensively explored in [12], and extended to the case of finite $\gamma > 1$. The original model of [2,3] and the variant pointed out in [11] appear as limiting $\gamma \to 0$ and $\gamma \to \infty$ cases, respectively. All these models [2,3,11,12] are defined by the same vertex, namely the square of the $SU(2)$ Wigner $15j$ symbol; they differ for the class of boundary states considered and their measure in the spinfoam sum. In [13], on the other hand, it was observed that the use of coherent states may not truly constraint the physical state space of the theory when the constraints are not entirely second class, and this happens in the limit case $\gamma \to \infty$. Therefore, while the coherent state technique introduced in [11] appears to work well in the $\gamma \to 0$ case, its straightforward extension to large γ yields a state space larger than the physical state space of LQG and—one might argue—larger that the proper quantum state space of gravity. Furthermore, the spectrum of the geometrical operators in this formulation turns out to be quite different from the standard one of loop quantum gravity [14]. Here, thus, we reconsider the finite γ case, but we solve the constraints using the same master constraint technique as in [2,3]. We leave the understanding of our results in terms of coherent states for future developments.

We find a model with a number of interesting properties. First, the second class constraints do reduce the dimension of the physical state space as one wants. Second, for all values of γ the state space precisely matches the one of LQG (on a fixed graph). This is particularly interesting in the case of the Lorentzian theory, where such a match is traditionally more problematic. Third, the spectrum of the area operator turns out to be discrete, and to be the same as in LQG, including the correct dependence on the Immirzi parameter γ. What is of particular interest is that this is true in the Lorentzian case as well, in spite of the fact that the unitary representations of the Lorentz group are labelled also by a continuous parameter. This provides a solution to a long-standing controversy: the area spectrum is discrete in LQG while it appears to be continuous in the spinfoam framework. The solution is that the area spectrum is continuous in spinfoams at the kinematical level, but it turns out to become discrete after proper implementation of the (second class) constraints. Finally, the *ad hoc* "flip" of the symplectic structure used to first derive the vertex in [2,3] is not required in the finite γ case.

All these developments rely on two basic ideas. The first, championed by Giorgio Immirzi [15], is to ("loop") quantize GR by first discretizing it on a Regge-like triangulation, with appropriately chosen variables. The second is to treat the simplicity constraints by first imposing them properly in a fixed $SO(4)$ (or $SO(3,1)$) gauge, and then projecting on the gauge invariant spaces. The implementation of these ideas is discussed in detail in [2,3]. Here, we briefly de-