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Nuclear Physics B 800 (2008) 349–383

NUCLEAR PHYSICS B

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Finite-size effects for dyonic giant magnons

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Received 16 January 2008; accepted 1 April 2008

Available online 16 April 2008

Abstract

We compute finite-size corrections to dyonic giant magnons in two ways. One is by examining the asymptotic behavior of helical strings of [K. Okamura, R. Suzuki, A perspective on classical strings from complex sine-Gordon solitons, Phys. Rev. D 75 (2007) 046001, hep-th/0609026] as elliptic modulus k goes to unity, and the other is by applying the generalized Lüscher formula for μ-term of [R.A. Janik, T. Łukowski, Wrapping interactions at strong coupling—the giant magnon, Phys. Rev. D 76 (2007) 126008, arXiv: 0708.2208 [hep-th]] to the situation in which incoming particles are boundstates. By careful choice of poles in the $su(2|2)^2$-invariant S-matrix, we find agreement of the two results, which makes possible to predict the (leading) finite-size correction for dyonic giant magnons to all orders in the 't Hooft coupling.

1. Introduction

There has been great advance toward understanding the correspondence between $\mathcal{N}=4$ super-Yang–Mills and superstring on $AdS_5 \times S^5$ recently. The AdS/CFT correspondence [1–3] predicts a map between individual string states and gauge invariant operators in super-Yang–Mills at least for large N, and under this map energy of a string state should be equal to conformal dimension of the corresponding operator.

Progress on checking this correspondence has been catalyzed by the discovery of integrability. The dilatation operator of $\mathcal{N}=4$ super-Yang–Mills theory is shown to have the same form as Hamiltonian of an integrable spin chain, which enables us to study the problem of diagonalizing the Hamiltonian by a technique called Bethe ansatz [4]. Long-range Bethe ansatz equations for the full $psu(2,2|4)$ sector are proposed to an arbitrary order of the 't Hooft coupling

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0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2008.04.007

$\lambda \equiv N g_{\mathrm{YM}}^2$ in [5], assuming all order integrability of super-Yang–Mills theory. Their original proposal contained so-called dressing phase, which was first introduced in [6]. The dressing phase reconciles mismatch between scaling limit of the Bethe ansatz equations of [7] and the integral equation derived from classical string theory [8]. An all-order expression of the dressing phase was later proposed in [9,10] on the assumptions of transcendentality [11–13] and crossing symmetry [14,15].

However, the long-range Bethe ansatz equations equipped with the dressing phase can reproduce the correct answer of super-Yang–Mills only when the length of spin chain L is large enough. For spin chains with finite size, the Bethe ansatz equations do not account for wrapping interactions [7], which possibly arise from the order of λ^L as higher-genus diagrams [16]. In fact, the Bethe ansatz prediction is found to disagree with the BFKL prediction [17–19] in [20]. Recently, it is found that the wrapping effects for the four-loop anomalous dimensions of certain short operators induce terms of higher degrees of transcendentality [21,22].

The wrapping problem does not occur for the system of infinite L at weak coupling, and such situation has been studied under the name of asymptotic spin chain [23,24]. It was shown that the S-matrix of the asymptotic spin chain can be determined only by the symmetry algebra $psu(2|2)^2 \ltimes \mathbb{R}^3$ up to the dressing phase, and that its BPS relation constrains the dispersion relation of magnon excitations as $\varepsilon(p) = [1 + f(\lambda)\sin^2(\frac{p}{2})]^{1/2}$, where the function $f(\lambda)$ is conjectured as the one given in (1.1). On string theory side, the asymptotic spin chain corresponds to the states with an infinite angular momentum. Classical string solutions which correspond to elementary magnon excitations over the asymptotic spin chain are found in [25] and called giant magnons. This correspondence is subsequently generalized to the one between magnon boundstates [26] and dyonic giant magnons [27].

The S-matrix of classical worldsheet theory on $AdS_5 \times S^5$ was examined in [28]. It was further found that the $psu(2|2)^2 \ltimes \mathbb{R}^3$ symmetry is realized in the worldsheet S-matrix when the level matching conditions are relaxed [29,30]. Moreover in [30], they proposed "string" S-matrix which satisfies the standard Yang–Baxter equation, while "gauge" S-matrix of [24] satisfies the twisted Yang–Baxter equation.

With remarkable success for the case of infinite L in mind, a natural question is what will be the dispersion relation of asymptotic spin chain when L is finite. From string theoretical point of view, answering to this question boils down to construction of classical string solutions with finite angular momenta which incorporate (dyonic) giant magnons.[1] Such solutions have already been constructed; see [31,32] for a general solution including giant magnons, and [33] for solutions including dyonic giant magnons. And it has been found in [31] that the energy-spin relation of giant magnons receives correction of the order e^{-cJ_1}, with J_1 the angular momentum along a great circle of $S^2 \subset S^5$ and $c = 2\pi/[\sqrt{\lambda}\sin(\frac{p}{2})]$.

It is argued in [34] that the exponential finite-size correction at strong coupling is related to the wrapping interaction at weak coupling, based on Thermodynamic Bethe ansatz approach [35–37] and the Lüscher formula [38–40]. Recently, Janik and Łukowski have elaborated this argument [41], assuming that Lüscher's argument can be applied to the non-relativistic dispersion relation

$$\varepsilon(p) = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\left(\frac{p}{2}\right)}. \tag{1.1}$$

[1] In conformal gauge, the "size" can be interpreted also as the circumference of worldsheet.