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Boundary ring: A way to construct approximate NG solutions with polygon boundary conditions I. Z_n-symmetric configurations

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Abstract

We describe an algebro-geometric construction of polygon-bounded minimal surfaces in AdS_5, based on the consideration of what we call the "boundary ring" of polynomials. The first non-trivial example of solutions to the Nambu–Goto (NG) equations for Z_6-symmetric hexagon is considered in some detail. Solutions are represented as power series, of which only the first terms are evaluated. The NG equations leave a number of free parameters (a free function). Boundary conditions, which fix the free parameters, are imposed on truncated series. A better use, albeit being exotic to theory of PDE, of the boundary ring is suggested as well. It is still unclear if explicit analytic formulas can be found in this way, but even approximate solutions, obtained by truncation of power series, can be sufficient to investigate the Alday–Maldacena—BDS/BHT version of the string/gauge duality.

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1. Introduction

1.1. BDS/BHT conjecture

One of the most important discoveries during the last few years of modern quantum field theory is the BDS conjecture [1], which—based on extensive investigations by many people during the last two decades—claims that the (MHV) amplitude of the n-gluon scattering in the planar limit of $N = 4$ SYM theory factorizes and exponentiates:

$$\mathcal{A}(\mathbf{p}_1, \ldots, \mathbf{p}_n|\lambda) = \mathcal{A}_{\text{tree}}\mathcal{A}_{\text{IR}}\mathcal{A}_{\text{finite}}, \tag{1.1}$$

where λ is the 't Hooft's coupling constant, $\mathcal{A}_{\text{tree}}$ and $\mathcal{A}_{\text{IR}}$ are the tree and IR-divergent amplitudes (the latter one is explicitly expressed through the celebrated anomalous dimension function $\gamma(\lambda)$—a subject of intensive but still unfinished research of the last years, an eigenvalue of a yet sophisticated integrable problem and a solution to an integral Bethe Ansatz equation [2]) and

$$\mathcal{A}_{\text{finite}} = \exp\left(\frac{1}{4}\gamma(\lambda)F_n^{(1)}(\mathbf{p}_1, \ldots, \mathbf{p}_n) + g_n(\lambda)\right), \tag{1.2}$$

where [3]

$$F_n^{(1)} = \oint_{\Pi}\oint \frac{dy^\mu\, dy'_\mu}{(y-y')^{2+\epsilon}}. \tag{1.3}$$

In this spectacular formula, Π is a polygon in the 4d Minkowski space with coordinates y_0, y_1, y_2, y_3, which is formed by n null vectors $\mathbf{p}_1, \ldots, \mathbf{p}_n$. The polygon is closed because of the energy–momentum conservation, $\mathbf{p}_1 + \cdots + \mathbf{p}_n = \mathbf{0}$. See [4] for a more detailed presentation of the BDS/BHT conjecture.

If BDS/BHT conjecture is true, it is the first exhaustive solution to a perturbative quantum field theory problem in 4 spacetime dimensions. Till today, it is constrained by a few restrictions:

- the theory has maximal supersymmetry ($N = 4$),
- only the planar limit is considered,
- only the MHV (maximal helicity violating) amplitudes are carefully analyzed,
- the answer is conjectured only for the scattering amplitudes, not for generic correlators of Wilson loops,
- there is no proof of the conjecture and there are even doubts that it is fully correct.

1.2. Alday–Maldacena conjecture

If the BDS conjecture is true, the amplitude should have the same momentum dependence in the strong-coupling regime. This means that the function $F_n^{(1)}(\mathbf{p}_1, \ldots, \mathbf{p}_n)$ should be also reproduced at the string side of the string/gauge (AdS/CFT) duality to all orders in the strong coupling expansion. In particular, since in the leading order it is given by a regularized minimal area of the world-sheet embedding into the AdS_5 space, one expects that

$$F_n^{(1)}(\mathbf{p}_1, \ldots, \mathbf{p}_n) \overset{(1.3)}{=} \oint_{\Pi}\oint \frac{dy^\mu\, dy'_\mu}{(y-y')^{2+\epsilon}} = \text{Minimal Area}_\epsilon, \tag{1.4}$$