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Massive Jackiw–Rebbi model

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Abstract

In this paper we analyze a generalized Jackiw–Rebbi (JR) model in which a massive fermion is coupled to the kink of the $\lambda\phi^4$ model as a prescribed background field. We solve this massive JR model exactly and analytically and obtain the whole spectrum of the fermion, including the bound and continuum states. The mass term of the fermion makes the potential of the decoupled second order Schrödinger-like equations asymmetric in a way that their asymptotic values at two spatial infinities are different. Therefore, we encounter the unusual problem in which two kinds of continuum states are possible for the fermion: reflecting and scattering states. We then show the energies of all the states as a function of the parameters of the kink, i.e. its value at spatial infinity (θ_0) and its slope at $x = 0$ (μ). The graph of the energies as a function of θ_0, where the bound state energies and the two kinds of continuum states are depicted, shows peculiar features including an energy gap in the form of a triangle where no bound states exist. That is, the zero mode exists only for θ_0 larger than a critical value (θ_0^c). This is in sharp contrast to the usual (massless) JR model where the zero mode and hence the fermion number $\pm 1/2$ for the ground state is ever present. This also makes the origin of the zero mode very clear: It is formed from the union of the two threshold bound states at θ_0^c, which is zero in the massless JR model.

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1. Introduction

In 1976 Jackiw and Rebbi [1] introduced the important concept of the fractional fermion number of the solitons, considering two different fermion-soliton models, one of them in one

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and the other in three spatial dimensions. In both models, the key observations that lead to the fractional charge of the soliton are that the models possess charge conjugation symmetry and also there is a nondegenerate zero-energy fermionic mode. They showed that in the presence of the zero mode the prescribed soliton is a degenerate doublet carrying charge $\pm 1/2$. In the ensuing decades there has been a vast number of works confirming and elaborating on the JR finding. This discovery has motivated much of the works on this subject and the concept of the vacuum polarization by background fields has been investigated in many branches of physics such as particle physics [1–12], cosmology [13–17], condensed-matter physics [18–21], polymer physics [22–24] and atomic physics [25–27].

We now explain some of these works which are relevant to this paper. In 1981 Goldstone and Wilczek [2] invented a powerful method, called the adiabatic method, for calculating the vacuum polarization of fermions induced by the background solitons. In this method the final topological background field, which is assumed to be slowly varying in space, is considered to be slowly evolving from the topologically trivial configuration. Using their method, they investigated some models which lack the symmetry in the energy spectrum of the fermion and showed that the fermion number of the vacuum can be any real number and not just $\pm 1/2$. Later on this method was generalized by MacKenzie and Wilczek [3,4]. In their method the requirement of the slow spatial variation of the background field was lifted and therefore they could consider models including solitons with arbitrary variations in the space. Using their method, they concluded that sharply varying solitons can never polarize the vacuum. Following these works, some authors used these methods to investigate the vacuum polarization for different models. In one of these papers, the authors [5] studied an exactly solvable model in which a fermion is coupled to a background field with two adjustable parameters. By varying these parameters, one can have different topological background fields with different topological charges and scale of variation. Using this simple model, they were able to explore the effect of the scale of variations of the solitons on the vacuum polarization.

In the JR model there is no explicit mass term for the Fermi field and the zero mode is always present, regardless of the values of the parameters of the model. These parameters are the Yukawa coupling constant, denoted by g, the values of the background field at spatial infinity, denoted by θ_0, and its slope at zero, denoted by μ. In a previous paper we presented exact solutions for the JR model and showed explicitly that there is a dynamically generated mass $M_0 = g\theta_0$ [28]. We also reasoned that as θ_0 increases and a mass gap appears in the spectrum, the two threshold bound states which separated the continua at $\theta_0 = 0$, join to form the ever present self-charge-conjugate nondegenerate zero mode. In this paper we generalize the JR model by adding an explicit mass term for the Fermi field, denoted by M, and solving the dynamical equations exactly, we find that the system possesses some unusual properties. In particular the potentials appearing in the two Schrödinger-like equations obtained from decoupling the Dirac equation have unequal values at $x \to \pm\infty$. Therefore, we have, in addition to the usual bound and continuum states, reflecting continuum states. Moreover, a schematic plot of the spectrum as a function of θ_0 reveals an energy gap region in the form of a triangle where no bound states can exist. The end point of this region is a critical value $\theta_0^c = M/g$. The zero mode is formed from the union of the threshold bound states present at this point and this zero mode exists for $\theta_0 > \theta_0^c$. For the JR model $\theta_0^c = 0$. Hence the vacuum polarization is zero for $\theta_0 \leqslant \theta_0^c$ and $\pm 1/2$ for $\theta_0 > \theta_0^c$.

In Section 2 we define the massive JR model which includes a massive fermion interacting with a prescribed background field in the form of the familiar kink. We then briefly discuss some important symmetries of our model which are the same as the original massless JR model. In Section 3 we obtain the second order decoupled Schrödinger-like equations obtained from