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ScienceDirect
Nuclear Physics B 795 (2008) 105–137
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

Heterotic twistor–string theory

Lionel Mason, David Skinner *

The Mathematical Institute, University of Oxford, 24-29 St. Giles, Oxford OX1 3LP, United Kingdom

Received 26 October 2007; received in revised form 8 November 2007; accepted 12 November 2007

Available online 19 November 2007

Abstract

We reformulate twistor–string theory as a heterotic string based on a twisted $(0,2)$ model. The path integral localizes on holomorphic maps, while the $(0,2)$ moduli naturally correspond to the states of $\mathcal{N}=4$ super-Yang–Mills and conformal supergravity under the Penrose transform. We show how the standard twistor–string formulae of scattering amplitudes as integrals over the space of curves in supertwistor space may be obtained from our model. The corresponding string field theory gives rise to a twistor action for $\mathcal{N}=4$ conformal supergravity coupled to super-Yang–Mills. The model helps to explain how the twistor–strings of Witten and Berkovits are related and clarifies various aspects of each of these models.

PACS: 11.25.Tq; 04.65.+e

Keywords: String theory; Twistor theory

1. Introduction

The twistor–string theories of Witten [1] and Berkovits [2] combine topological string theory with the Penrose transform [3] to describe field theories in four-dimensional spacetime. The models appear to be equivalent to each other and to $\mathcal{N}=4$ super-Yang–Mills theory coupled to a non-minimal conformal supergravity [4]. The mechanism is completely different from the usual string paradigm: spacetime is not introduced *ab initio* as a target, but emerges as the space of degree 1 worldsheet instantons in the twistor space target. It therefore provides a new way for both string theory and twistor theory to make contact with spacetime physics. As far as string theory is concerned, it does so without the extra spacetime dimensions and further infinite towers

* Corresponding author.
E-mail address: dskinner@perimeterinstitute.ca (D. Skinner).

doi:10.1016/j.nuclphysb.2007.11.010

of massive modes of conventional string theory. As far as twistor theory is concerned, it resolves (albeit perturbatively) the most serious outstanding questions in the twistor programme. Firstly, it provides a solution to the 'googly problem' of encoding both the selfdual and anti-selfdual parts of Yang–Mills and gravitational fields on twistor space in such a way that interactions can be naturally incorporated. Classical twistor constructions have previously only been able to cope with anti-selfdual interactions. Secondly, twistor–string theory also provides a natural way to incorporate quantum field theory into twistor theory. Moreover the associated twistor–string field theory is closely related to the twistor actions constructed in [5–7]. These actions provide generating principles for all the amplitudes in the theories. Insight from the twistor–string has also led to a number of powerful new approaches to calculating scattering amplitudes in perturbative gauge theory, both directly in string theory [8–10], and indirectly through spacetime unitarity methods inspired by the twistor–string [11–17].

There remain a number of difficulties in making sense of twistor–string theory, and in exploiting it as a calculational tool. In particular, the presence of conformal supergravity limits ones ability to use twistor–string theory to calculate pure Yang–Mills amplitudes to tree level, since supergravity modes will propagate in any loops [1,18]. Conformal supergravity is thought neither to be unitary, nor to possess a stable vacuum [19] and so is widely viewed as an unwelcome feature of twistor–string theory. However, because conformal supergravity contains Poincaré supergravity as a subsector, one might more optimistically view it as an opportunity. Indeed, twistor–string theories with the spectrum of Poincaré supergravity have been constructed in [20], although these theories remain tentative as it has not yet been determined whether they lead to the correct interactions. If they do, and are consistent, they will provide a new approach to quantum gravity. Furthermore, for applications to loop calculations in gauge theories, one might then decouple gravity in the limit that the Planck mass becomes infinite while the gauge coupling stays finite.

This paper will not attempt to make further progress on these issues, but will provide a new model for twistor–string theory that goes some way towards resolving other puzzles arising from the original models. Witten's original twistor–string [1] is based on a topological string theory, the B-model, of maps from a Riemann surface into the twistor superspace $\mathbb{P}^{3|4}$, the projectivization of $\mathbb{C}^{4|4}$ with four bosonic coordinates and four fermionic. While one can always construct a topological string theory on a standard (bosonic) Calabi–Yau threefold [21,22], it is not obvious that the same construction works on a supermanifold such as $\mathbb{P}^{3|4}$ even if it is formally Calabi–Yau. Proceeding heuristically, Witten showed that the open string sector would successfully provide the anti-selfdual[1] interactions of $\mathcal{N}=4$ super-Yang–Mills. However, to include selfdual interactions requires the introduction of D1 branes wrapping holomorphic curves in projective supertwistor space. The full Yang–Mills perturbation theory then arises from strings stretched between these D1 branes and a stack of (almost) space-filling D5 branes, together with the holomorphic Chern–Simons theory of the D5–D5 strings. However, one would also expect to find open D1–D1 strings and the role of these in spacetime was left unclear. Gravitational modes decouple from the open B-model at the perturbative level, so conformal supergravity arises through the dynamics of the D1 branes in a manner that was not made entirely transparent. These D branes are non-perturbative features of the B-model and thus to fully understand the presence of conformal supergravity in Witten's model (perhaps so as to explore related theories with Einstein gravity), one would appear to have to understand the full non-perturbative topolog-

1 Our conventions are those of Penrose and Rindler [23], whereby an on-shell massless field of helicity h is represented on twistor space $\mathbb{PT}'$ by an element of $H^1(\mathbb{PT}', \mathcal{O}(-2h-2))$; these conventions differ from those of Witten [1].