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Conformal properties of four-gluon planar amplitudes and Wilson loops

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Abstract

We present further evidence for a dual conformal symmetry in the four-gluon planar scattering amplitude in $\mathcal{N}=4$ SYM. We show that all the momentum integrals appearing in the perturbative on-shell calculations up to four loops are dual to true conformal integrals, well defined off shell. Assuming that the complete off-shell amplitude has this dual conformal symmetry and using the basic properties of factorization of infrared divergences, we derive the special form of the finite remainder previously found at weak coupling and recently reproduced at strong coupling by AdS/CFT. We show that the same finite term appears in a weak coupling calculation of a Wilson loop whose contour consists of four light-like segments associated with the gluon momenta. We also demonstrate that, due to the special form of the finite remainder, the asymptotic Regge limit of the four-gluon amplitude coincides with the exact expression evaluated for arbitrary values of the Mandelstam variables.

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1. Introduction

Gluon and quark scattering amplitudes have long been the subject of numerous studies in QCD. These amplitudes have a very non-trivial structure consisting of infrared singular and finite parts. In physical infrared safe observables like inclusive cross-sections the former cancel in the sum of all diagrams [1] while the latter produce a rather complicated function of the kinematic variables. The infrared singular part of the scattering amplitude has a universal structure [2–4] which is intimately related to the properties of Wilson loops [5]. This leads to an evolution equation for the amplitude as a function of the infrared cutoff [6–8] which is governed, in the planar limit, by the so-called cusp anomalous dimension of the Wilson loop [9,10]. This anomalous dimension first emerged in the studies of *ultraviolet* cusp singularities of Wilson loops [11] (see also [12] and references therein) and its relation to *infrared* asymptotics in gauge theories was discovered in [5,13,14]. The cusp anomalous dimension is very important in QCD since it controls the asymptotic behavior of various gauge invariant quantities like the double-log (Sudakov) asymptotics of form factors, the logarithmic scaling of the anomalous dimension of higher-spin operators, the gluon Regge trajectory, etc. However, unlike the singular part, the finite part of the gluon scattering amplitude in QCD is much more involved, being given in terms of certain special functions of the Mandelstam variables.

Recently, a lot of attention has been paid to the problem of calculating gluon scattering amplitudes in the context of the maximally supersymmetric Yang–Mills theory ($\mathcal{N}=4$ SYM). These amplitudes have been extensively studied in perturbation theory where they have been constructed using state-of-the-art unitarity cut techniques [15–19]. The results of these studies concern both the divergent and finite parts of the amplitude. Although the main subject of the present paper is the finite part, we start with a brief review of the IR singularities.

1.1. Infrared divergences in gluon scattering amplitudes

Unlike a generic gauge theory, $\mathcal{N}=4$ SYM is ultraviolet finite. Despite this UV finiteness, the gluon scattering amplitudes are still IR divergent, even though the singular structure is much simpler compared to QCD, due to the fact that the coupling does not run. As in QCD, the dependence on the IR cutoff is determined by the cusp anomalous dimension.

The notion of cusp anomalous dimension was initially introduced [11,12] in the context of a Wilson loop evaluated over a closed (Euclidean) contour with a cusp (see Fig. 1). By definition, $\Gamma_{\rm cusp}(a,\vartheta)$ is a function of the coupling constant a and the cusp angle ϑ describing the dependence of the Wilson loop on the *ultraviolet* cutoff. Later on it was realized [5,13] that the same quantity $\Gamma_{\rm cusp}(a,\vartheta)$ determines the *infrared* asymptotics of scattering amplitudes in gauge theories, for which a dual Wilson loop is introduced with an integration contour C uniquely defined by the particle momenta. The cusp angle ϑ is related to the scattering angles and it takes large values in Minkowski space, $|\vartheta|\gg 1$. In this limit, $\Gamma_{\rm cusp}(a,\vartheta)$ scales linearly in ϑ to all loops [14]

$$\Gamma_{\rm cusp}(a,\vartheta)=\vartheta\,\Gamma_{\rm cusp}(a)+O\big(\vartheta^0\big), \tag{1}$$

where $\Gamma_{\rm cusp}(a)$ is a function of the coupling constant only. In what follows we shall use the term *cusp anomalous dimension* in this restricted sense, to denote the quantity $\Gamma_{\rm cusp}(a)$. In a dimensionally regularized four-gluon scattering amplitude, the IR poles exponentiate and $\Gamma_{\rm cusp}(a)$ controls the coefficient of the double pole in the exponent.