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Emptiness formation probability in the domain-wall six-vertex model

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Abstract

The emptiness formation probability in the six-vertex model with domain wall boundary conditions is considered. This correlation function allows one to address the problem of limit shapes in the model. We apply the quantum inverse scattering method to calculate the emptiness formation probability for the inhomogeneous model. For the homogeneous model, the result is given both in terms of certain determinant and as a multiple integral representation.

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1. Introduction

A special kind of fixed boundary conditions, the so-called domain wall boundary conditions, was first considered for the six-vertex model by Korepin in seminal paper [1]. In [2], Izergin showed that the partition function of the model on the finite lattice can be found exactly in terms of certain determinant; see also paper [3] for details. It was later shown in [4,5], by studying the thermodynamic limit, that the free energy per site is different with respect to the case of periodic

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boundary conditions. This fact hints at spatial separation of phases (e.g., ferroelectric order and disorder), which is confirmed both numerically [6–8] and analytically [9,10].

To get some details about the phase separation phenomena, e.g., to find the shape of the spatial curve separating the phases, or limit shape, one has to know some appropriate correlation function. The problem of computing the correlation functions in the 'domain-wall' six-vertex model has been addressed in papers [11–14] where some correlation functions near the boundary were found.

In this paper, we continue the study of correlation functions of the six-vertex model with domain wall boundary conditions. Specifically, we consider here a particular non-local correlation function, the emptiness formation probability (EFP). This function describes the probability of having a set of consecutive horizontal edges along a given column, all in a given state; we consider here the case when the set starts from the top boundary and extends inside the lattice. This correlation function allows one to address the problem of limit shapes in the model [15].

To compute EFP, we follow the lines of papers [12,14,16] where the quantum inverse scattering method (QISM) [17,18] and some facts from the theory of orthogonal polynomials were used. Mostly following ideas (as well as notations) of [14], we represent EFP in certain determinantal form, which is shown here to be also equivalent to some multiple integral. This last representation recalls analogous multiple integral representations for correlation functions of quantum spin chains [19–22].

The paper is organized as follows. In Section 2 we start with giving some definitions and fixing some notations. The quantum inverse scattering method in application to the model is considered in Section 3. The core calculation of EFP for the inhomogeneous model is contained in Section 4. The homogeneous limit is performed in Section 5, where the main result is given both in terms of certain determinant and as a multiple integral representation. Section 6 is devoted to discussion of equivalent multiple integral representations for EFP.

2. Some definitions and notations

2.1. The model

The six-vertex model is a statistical mechanics model in which the local states are associated with edges of a square lattice, and the Boltzmann weights are assigned to its vertices. The states can take two values, which are often denoted by arrows pointing along the edge. Among the sixteen possible arrow configurations around a vertex only six are allowed (having nonzero Boltzmann weights), with equal number of incoming and outgoing arrows. In this paper we consider the model on a lattice having both N rows and N columns ('the $N \times N$ lattice') with the boundary states fixed in a special way: all arrows on the left and right boundaries are outgoing while on the top and bottom boundaries all arrows are incoming. Such a model is called the six-vertex model with domain wall boundary conditions.

In the six-vertex model with invariance under reversal of all arrows there are three possible values for Boltzmann weights at each vertex, usually denoted as a, b, and c. To use the quantum inverse scattering method (QISM) for calculations we will consider the inhomogeneous version of the model, in which the weights of the vertex being at the intersection of kth horizontal line and αth vertical line are

$$a_{\alpha k} = a(\lambda_\alpha, \nu_k), \qquad b_{\alpha k} = b(\lambda_\alpha, \nu_k), \qquad c_{\alpha k} = c, \tag{2.1}$$