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The QCD spin chain S matrix

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Abstract

Beisert et al. have identified an integrable $SU(2,2)$ quantum spin chain which gives the one-loop anomalous dimensions of certain operators in large N_c QCD. We derive a set of nonlinear integral equations (NLIEs) for this model, and compute the scattering matrix of the various (in particular, magnon) excitations.

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1. Introduction

The search for integrability in QCD has a long history (see, e.g., [1–6] and references therein). A remarkable recent development is the discovery [7] that the one-loop mixing matrix[1] for the chiral gauge-invariant operators

$$\mathrm{tr}\, f_{\alpha_1\beta_1}(x)\ldots f_{\alpha_L\beta_L}(x) \tag{1.1}$$

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[1] Given a set of operators $\mathcal{O}^M(x)$, the mixing matrix is defined by $\Gamma = Z^{-1}\cdot dZ/d\ln\Lambda$, where Z is the renormalization factor which makes correlation functions of $\mathcal{O}^M_{\mathrm{ren}}(x) = Z^M_N\mathcal{O}^N(x)$ finite, and Λ is the ultraviolet cutoff. See also [8].

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in the limit $N_c \to \infty$ is given by the integrable spin-1 antiferromagnetic XXX Hamiltonian [9,10],

$$\Gamma = \frac{\alpha_s N_c}{2\pi} \sum_{l=1}^{L} \left[\frac{7}{6} + \frac{1}{2} \vec{S}_l \cdot \vec{S}_{l+1} - \frac{1}{2} (\vec{S}_l \cdot \vec{S}_{l+1})^2 \right]. \tag{1.2}$$

Here $f_{\alpha\beta}$ are the selfdual components of the Yang–Mills field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_{\text{YM}}[A_\mu, A_\nu]$ (where the gauge fields $A_\mu(x)$ are $N_c \times N_c$ Hermitian matrices), which together with the anti-selfdual components $\bar{f}_{\dot\alpha\dot\beta}$ are defined by

$$F_{\mu\nu} = \sigma_{\mu\nu}^{\alpha\beta} f_{\alpha\beta} + \bar{\sigma}_{\mu\nu}^{\dot\alpha\dot\beta} \bar{f}_{\dot\alpha\dot\beta}, \tag{1.3}$$

where $\sigma_{\mu\nu} = i\sigma_2(\sigma_\mu \bar{\sigma}_\nu - \sigma_\nu \bar{\sigma}_\mu)/4$, $\bar{\sigma}_{\mu\nu} = -i(\bar{\sigma}_\mu \sigma_\nu - \bar{\sigma}_\nu \sigma_\mu)\sigma_2/4$ and $\sigma_\mu = (1, \vec{\sigma})$, $\bar{\sigma}_\mu = (1, -\vec{\sigma})$. Moreover, $\alpha_s = g_{YM}^2/4\pi$, $\alpha_s N_c$ is the 't Hooft coupling [1] which is assumed to be small, and $\vec{S}$ are spin-1 generators of $SU(2)$. Indeed, since $f_{\alpha\beta}$ has three independent components

$$f_+ = f_{11}, \qquad f_0 = \frac{1}{\sqrt{2}}(f_{12} + f_{21}), \qquad f_- = f_{22}, \tag{1.4}$$

the operators (1.1) can be identified with the Hilbert space of a periodic spin-1 quantum spin chain of length L. The eigenvectors and eigenvalues of Γ, i.e., the linear combinations of the operators (1.1) which are multiplicatively renormalizable and their anomalous dimensions, respectively, can therefore be obtained using the Bethe ansatz [11,12]. In particular, the anomalous dimensions are given by

$$\gamma = \frac{\alpha_s N_c}{2\pi} \left(\frac{7L}{6} - \sum_{j=1}^{M_l} \frac{2}{l_j^2 + 1} \right), \tag{1.5}$$

where $\{l_1, \ldots, l_{M_l}\}$ are roots of the Bethe ansatz equations (BAEs)[2]

$$\left(\frac{l_j + i}{l_j - i} \right)^L = \prod_{\substack{k=1 \\ k \neq j}}^{M_l} \frac{l_j - l_k + i}{l_j - l_k - i}. \tag{1.6}$$

This result was generalized in [13] to gauge-invariant operators with derivatives

$$\operatorname{tr}\big(D^{m_1} f\big) \ldots \big(D^{m_L} f\big), \tag{1.7}$$

where

$$D^m f = D_{\alpha_1 \dot\alpha_1} \cdots D_{\alpha_m \dot\alpha_m} f_{\beta\gamma} + \text{symmetrized} \tag{1.8}$$

(complete symmetrization in the undotted and dotted indices, respectively), and $D_\mu = \sigma_\mu^{\alpha\dot\alpha} D_{\alpha\dot\alpha}$ is the usual Yang–Mills covariant derivative. Namely, the one-loop mixing matrix for the operators (1.7) is given by an integrable $SO(4,2) = SU(2,2)$ (non-compact!) quantum spin chain Hamiltonian with spins in the representation with Dynkin labels $[2, -3, 0]$. The anomalous dimensions are given by

$$\gamma = \frac{\alpha_s N_c}{2\pi} \left(\frac{7L}{6} - \sum_{j=1}^{M_l} \frac{2}{l_j^2 + 1} + \sum_{j=1}^{M_u} \frac{3}{u_j^2 + 9/4} \right), \tag{1.9}$$

[2] There is an additional (zero-momentum) equation due to the cyclicity of the trace in the operators.