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The bases of effective field theories

Martin B. Einhorn [a,b,*], José Wudka [c]

[a] *Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, United States*
[b] *Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109, United States*
[c] *Department of Physics and Astronomy, University of California, Riverside, CA 92521-0413, United States*

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Abstract

With reference to the equivalence theorem, we discuss the selection of basis operators for effective field theories in general. The equivalence relation can be used to partition operators into equivalence classes, from which inequivalent basis operators are selected. These classes can also be identified as containing Potential-Tree-Generated (PTG) operators, Loop-Generated (LG) operators, or both, independently of the specific dynamics of the underlying extended models, so long as it is perturbatively decoupling. For an equivalence class containing both, we argue that the basis operator should be chosen from among the PTG operators, because they may have the largest coefficients. We apply this classification scheme to dimension-six operators in an illustrative Yukawa model as well in the Standard Model (SM). We show that the basis chosen by Grzadkowski et al. [5] for the SM satisfies this criterion. In this light, we also revisit and verify our earlier result [6] that the dimension-six corrections to the triple-gauge-boson couplings only arise from LG operators, so the magnitude of the coefficients should only be a few parts per thousand of the SM gauge coupling if BSM dynamics respects decoupling. The same is true of the quartic-gauge-boson couplings.

Keywords: Effective Lagrangians; Beyond Standard Model; Effective field theory

1. Introduction

Effective quantum field theories have a wide variety of applications in condensed matter [1] and elementary particle physics [2,3], both as methods for facilitating calculations and as ways of

* Corresponding author at: Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030, United States

E-mail address: meinhorn@kitp.ucsb.edu (M.B. Einhorn).

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exploring or constraining new physics. In this last application, there has been a revival of interest among high-energy physicists since the discovery of a Higgs boson at the CERN LHC, apparently resolving the long-standing uncertainty about the theory of elementary particles known as the Standard Model (SM).

Without knowing the precise form of new degrees of freedom or new particles, there are inherent ambiguities in the form of additional operators to be added to a theory because of the equivalence theorem. As reviewed in more detail below, this states that observable transition amplitudes (S-matrix elements) are unchanged by replacing some operators with others if their difference vanishes "on-shell", *i.e.*, if the difference vanishes when the classical equations of motion (EoM) are satisfied. Although this allows one to reduce the number of new operators and coupling constants that must be introduced [4,5], in the face of ignorance of the underlying dynamics, it seems the choices are both arbitrary and irrelevant.

In any given model extending the SM to higher energy scales, some operators Q_i may arise from tree diagrams in an underlying theory, while others may only emerge from loop corrections. Generally, the coefficients of loop diagrams, as quantum corrections to the classical theory, are perturbatively smaller than those associated with tree diagrams, being associated with higher powers of dimensionless coupling constants times factors of $(16\pi^2)^{-n}$, where n is the number of loops. Even in nonperturbative applications [3], such distinctions between trees and loops can be important although less so because of the strong interactions that are involved. It has been observed [6], however, that symmetries associated with the known dynamics, when preserved by the underlying new degrees of freedom, may be used to classify operators arising from the underlying dynamics, irrespective of the particular model.

Given that there is some arbitrariness in the choice of operators, there has been a good deal of recent discussion concerning the "best" choice to make to perform fits to experimental data [7,8]. A number of people have cited the difficulties deciding among equivalent operators, because some arise in tree approximation while others may arise only from loop diagrams [9,5].

In the past, it has been argued (*e.g.*, in Ref. [9]) that, because the equivalence theorem relates some operators arising from loops to operators arising from trees, there is no way to decide *a priori* which operators to choose. In this paper, we shall discuss the best way to choose among such higher-order operators. The inherent ambiguities discussed in Ref. [9] can be unraveled in a general way, independent of any particular application. We shall limit our discussion to perturbative applications,[1] where such distinctions are most important, although perhaps this could be extended to other applications. Elsewhere [10], we shall apply this the potential influence of physics beyond the Standard Model (BSM) to the determination of the properties of the observed Higgs boson. This is what inspired the present investigation which, in the end, led to conclusions quite independent of that motivation.

An outline of the paper is as follows: In the next section, we review some features of effective Lagrangians, with particular attention the equivalence theorem. In Appendix A, we discuss some technical complications associated with masses and superrenormalizable couplings. In Section 3, we explain how the equivalence theorem can be used as an equivalence relation to partition the set of operators. In Appendix B, we illustrate these concepts in a simple Yukawa model. In Section 4, we explain that operators may be classified as Potential-Tree-Generated (PTG) operators or Loop-Generated (LG) operators, irrespective of the underlying model or theory, and advocate choosing as basis vectors PTG operators to the extent possible. In Section 5 and Appendix C

[1] We will assume that we are dealing with relativistic quantum fields in four dimensions.