Available online at www.sciencedirect.com
ScienceDirect
Nuclear Physics B 797 [PM] (2008) 431–463
NUCLEAR PHYSICS B
www.elsevier.com/locate/nuclphysb

One-loop corrections to the mass of self-dual semi-local planar topological solitons

A. Alonso Izquierdo [a], W. Garcia Fuertes [b], M. de la Torre Mayado [c], J. Mateos Guilarte [d,*]

[a] *Departamento de Matematica Aplicada, Universidad de Salamanca, Spain*
[b] *Departamento de Fisica, Universidad de Oviedo, Spain*
[c] *Departamento de Fisica, Universidad de Salamanca, Spain*
[d] *Departamento de Fisica and IUFFyM, Universidad de Salamanca, Spain*

Received 10 September 2007; received in revised form 16 November 2007; accepted 19 November 2007

Available online 28 November 2007

Abstract

A formula is derived that allows the computation of one-loop mass shifts for self-dual semilocal topological solitons. These extended objects, which in three spatial dimensions are called semi-local strings, arise in a generalized Abelian Higgs model with a doublet of complex Higgs fields. Having a mixture of global, $SU(2)$, and local (gauge), $U(1)$, symmetries, this weird system may seem bizarre, but it is in fact the bosonic sector of electro-weak theory when the weak mixing angle is $\frac{\pi}{2}$. The procedure for computing the semi-classical mass shifts is based on canonical quantization and heat kernel/zeta function regularization methods.

PACS: 03.70.+k; 11.15.Kc; 11.15.Ex

Keywords: Semi-local topological solitons; Heat-kernel/zeta function regularization; One-loop shifts to soliton masses

1. Introduction

The purpose of this paper is to address the computation and analysis of one-loop shifts to the masses of semi-local planar topological solitons arising in a natural generalization of the Abelian

* Corresponding author.
E-mail address: guilarte@usal.es (J.M. Guilarte).

0550-3213/$ – see front matter
doi:10.1016/j.nuclphysb.2007.11.023

Higgs model. Seen in $(3+1)$-dimensional space–time these solitons become semi-local strings whereas their masses give the string tensions, see [1,2]. At the critical point that marks the phase transition between type I and type II superconductivity, the set of semi-local self-dual topological solitons is an interesting $4l$-dimensional moduli space, where l is the number of quanta of the magnetic flux, see [3,4]. Recently, superconducting semi-local (non-self-dual) strings with very intriguing properties have been discovered in this model [5].

Computations of one-loop mass corrections will be performed using the heat kernel/zeta function regularization method. The high-temperature asymptotic expansion of the heat function, see [6–9], is a powerful tool that was applied for the first time to the calculation of kink mass shifts in [10] (the $\mathcal{N}=1$ SUSY case) and [11] (the non-SUSY case). One-loop corrections to $\mathcal{N}=2$ supersymmetric self-dual Nielsen–Olesen vortices were computed in a similar approach by Vassilevich and Rebhan–van Nieuwenhuizen–Wimmer in Refs. [12] and [13]. In the second paper, the authors also showed that the central charge of the SUSY algebra is modified in one-loop order in such a way that the Bogomolny bound is saturated at the semi-classical level. Sometime later, we calculated the one-loop mass shift for $\mathcal{N}=0$ (non-SUSY) self-dual NO vortices carrying a quantum of magnetic flux in Ref. [15]. Mass shifts have been given for spherically symmetric self-dual vortices (when several solitons of a quantum of flux have coinciding centers) in [16] up to four magnetic flux quanta. Cruder approximations were also provided for the mass shift of two separated self-dual NO vortices—each of them with a quantum of magnetic flux—as a function of the inter-center distance.

In Ref. [17], we studied the one-loop correction to the energy of a degenerate manifold of kinks that arise in a very interesting family of models with two real scalar fields in $(1+1)$ dimensions. These field theoretical systems are obtained through dimensional reduction—plus a reality condition—of an $\mathcal{N}=1$ supersymmetric Wess–Zumino model with two chiral superfields, see [18–20]. A comparison between the mass shifts of these composite kinks and the correction to the mass of the ordinary $\lambda\phi_2^4$ kink was offered in [21]. In this paper, we address a similar, but more difficult, situation in $(2+1)$ dimensions, comparing one-loop mass corrections of the topological solitons that arise in two planar Abelian gauge systems; one with two complex scalar fields and the other with a single complex scalar field. The methodology used to accomplish this task is explained in detail in Ref. [22], where a complete list of references can be found.

Our paper is organized as follows: in Section 2 we describe the model and develop perturbation theory around one of the vacua. A one-loop renormalization is also performed. Section 3 is devoted to summarizing the structure of the moduli space of self-dual topological solitons. As a novelty, we also apply a variation of the de Vega–Shaposnik method [23] to find numerical solutions for spherically symmetric topological solitons. In Section 4, we explain how to obtain one-loop mass shifts in terms of generalized zeta functions of the second-order differential operators ruling the small fluctuations of the bosonic and ghost fields, and in Section 5 the high-temperature expansion of the heat kernel is used to give the final formula for one-loop mass shifts of semi-local self-dual topological solitons after application of Mellin's transforms. We present our results in Section 6 by means of Mathematica calculations of the coefficients of the asymptotic series giving the heat functions. Finally, in Appendix A we offer Tables A.1–A.5 where these coefficients are shown.