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The strong coupling limit of the scaling function from the quantum string Bethe ansatz

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Abstract

Using the quantum string Bethe ansatz we derive the one-loop energy of a folded string rotating with angular momenta (S, J) in $AdS_3 \times S^1 \subset AdS_5 \times S^5$ in the limit $1 \ll J \ll S$, $z = \sqrt{\lambda}\log(S/J)/(\pi J)$ fixed. The one-loop energy is a sum of two contributions, one originating from the Hernandez–Lopez phase and another one being due to spin chain finite size effects. We find a result which at the functional level exactly matches the result of a string theory computation. Expanding the result for large z we obtain the strong coupling limit of the scaling function for low twist, high spin operators of the $SL(2)$ sector of $\mathcal{N} = 4$ SYM. In particular, we recover the famous $-\frac{3\log(2)}{\pi}$. Its appearance is a result of non-trivial cancellations between the finite size effects and the Hernandez–Lopez correction.

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1. Introduction

Due to recent years discovery of integrable models underlying the spectral problems of both $\mathcal{N} = 4$ SYM [1] and type IIB string theory on $AdS_5 \times S^5$ [2] the spectral part of the AdS/CFT conjecture [3] can now be stated in a very pointed manner. Namely, the conjecture simply says

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that the S-matrix of the respective integrable models must agree [4]. Furthermore, the common symmetry group of the two theories constrains the S-matrix up to a phase factor [5]. The formulation of the conjecture can thus be further sharpened to the statement that the phase factors of respectively $\mathcal{N}=4$ SYM and type IIB string theory on $AdS_5 \times S^5$ should be identical.

Based on educated guessing, phase factors for both $\mathcal{N}=4$ SYM and type IIB string theory on $AdS_5 \times S^5$ have been put forward. In accordance with the strong-weak coupling nature of the AdS/CFT correspondence the gauge theory phase factor [6] is given as an infinite series in the 't Hooft coupling constant λ whereas the string theory phase factor [7] is given as an asymptotic expansion in $1/\sqrt{\lambda}$. There exist arguments that the string theory asymptotic expansion for large λ can originate from the same function as defined by the gauge theory perturbative expansion which has a finite radius of convergence [6]. However, both phase factors are rather involved functions and it would be reassuring to see an example of a simple observable which can be extrapolated smoothly from weak to strong coupling.

A candidate for such an observable is the universal scaling function or cusp anomalous dimension, $f(g)$ where $g^2 = \frac{\lambda}{8\pi^2}$. It is related to the anomalous dimension of low twist operators of $\mathcal{N}=4$ SYM of the type

$$\mathcal{O} = \mathrm{Tr}\big(D^S Z^J + \cdots\big). \tag{1}$$

Here D is a light cone covariant derivative, Z is a complex scalar, S is the space–time spin and J is denoted as the twist. For leading twist, i.e. $J=2$, it is well known that the anomalous dimension Δ of such an operator for large values of the spin grows logarithmically with the spin

$$\Delta - S = f(g)\log(S), \quad S \to \infty, \tag{2}$$

where $f(g)$ can be expanded perturbatively in g. The scaling function has the appealing feature that, as opposed to other observables one could think of, it depends only on one parameter g. For instance, it is not polluted by any additional J-dependence. The function $f(g)$ has been determined by solid field theory calculations up to and including four-loop order [8]. Furthermore, starting from the asymptotic gauge theory Bethe equations [9], inserting the conjectured gauge theory phase factor [6] and taking a large-S limit it has been possible to derive an equation which determines $f(g)$ to all orders in g [10]. This equation, known as the BES equation, correctly reproduces the known first four orders in g^2. Its derivation, however, relies on the assumption that the scaling function is the same for all operators with a *finite* value of the twist and that at the same time it is permitted to take J sufficiently large so that the asymptotic Bethe equations are correct.

On the string theory side a low twist, high spin operator corresponds to a folded string rotating with angular momentum S on $AdS_3 \subset AdS_5 \times S^5$ [11]. The energy of such a string has an expansion for large λ which reads

$$E = \left(\frac{\sqrt{\lambda}}{\pi} - \frac{3\log(2)}{\pi} + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right)\right)\log S, \quad S \to \infty. \tag{3}$$

Here the first term follows from semi-classical analysis [11] and the second one from a one-loop computation [12]. Deriving this result from the Bethe equations would yield a very comforting confirmation of both the integrability approach as well as of the AdS/CFT conjecture itself. However, the strong coupling analysis of the BES equation has proved hard. For the moment only the leading semi-classical contribution has been derived from the BES equation by analytic means [13–15]. By numerical analysis of the equation both the leading [16,17] and the next to