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Strong-coupling expansion of cusp anomaly and gluon amplitudes from quantum open strings in $AdS_5 \times S^5$

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Abstract

An important "observable" of planar $\mathcal{N} = 4$ SYM theory is the scaling function $f(\lambda)$ that appears in the anomalous dimension of large spin twist 2 operators and also in the cusp anomaly of light-like Wilson loop. The non-trivial relation between the anomalous dimension and the Wilson loop interpretations of $f(\lambda)$ is well-understood on the perturbative gauge theory side of the AdS/CFT duality. In the first part of this paper we present the dual string-theory counterpart of this relation, i.e., the equivalence between the closed-string and the open-string origins of $f(\lambda)$. We argue that the coefficient of the $\log S$ term in the energy of the closed string with large spin S in AdS_5 should be equal to the coefficient in the logarithm of expectation value of the null cusp Wilson loop, to all orders in $\lambda^{-1/2}$ expansion. The reason is that the corresponding minimal surfaces happen to be related by a conformal transformation (and an analytic continuation). As a check, we explicitly compute the leading 1-loop string sigma model correction to the cusp Wilson loop, reproducing the same subleading coefficient in $f(\lambda)$ as found earlier in the spinning closed string case. The same function $f(\lambda)$ appears also in the resummed form of the 4-gluon amplitude as discussed at weak coupling by Bern, Dixon and Smirnov and recently found at the leading order at strong coupling by Alday and Maldacena (AM). Here we attempt to extend the latter approach to a subleading order in $\lambda^{-1/2}$ by computing the IR singular part of the 1-loop string correction to the corresponding T-dual Wilson loop. We

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discuss explicitly the 1-cusp case and comment on apparent problems with the dimensional regularization proposal of AM when directly applied order by order in strong coupling (string inverse tension) expansion.

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1. Introduction

As is well known, in perturbative (planar $\mathcal{N}=4$) gauge theory there are two alternative routes that lead to the same scaling function $f(\lambda)$: it can be found as a coefficient in the anomalous dimension of gauge-invariant large spin twist two operator [1,2] or as a cusp anomaly of a light-like Wilson line [3–6]. One can give a general proof of the equivalence between the two pictures in perturbative gauge theory [5].

On the dual perturbative $AdS_5 \times S^5$ string theory side the anomalous dimension of minimal twist operator is represented by the energy of a *closed* string with large spin $S \gg 1$ in AdS_5 [7], $E = S + f(\lambda)\ln S + \cdots$, $f(\lambda)_{\lambda\gg 1} = \frac{\sqrt{\lambda}}{\pi} + \cdots$. The same result for the strong-coupling limit of $f(\lambda)$ was shown in [8] (see also [9]) to follow from the *open* string picture, i.e., from the area of a surface ending on a cusp formed by two light-like Wilson lines on the boundary of AdS_5.

The definitions of $f(\lambda)$ in [7] and [8] seem very different and a priori unrelated, in contrast to the known perturbative gauge theory equivalence of the anomalous dimension of large spin twist 2 operator and the Wilson loop cusp anomaly function. In particular, while it was possible to compute the two subleading quantum corrections to $f(\lambda)$ in the closed spinning string picture [10–12][2]

$$f(\lambda)_{\lambda\gg 1} = a_0\sqrt{\lambda} + a_1 + \frac{a_2}{\sqrt{\lambda}} + \cdots, \qquad a_0 = \frac{1}{\pi},\ a_1 = -\frac{3}{\pi}\ln 2, \tag{1.1}$$

the direct computation of the quantum string corrections in the Wilson loop approach [8,13,14] appeared to be harder (for previous attempts in that direction see [15–17]).

One of our aims below will be to explain the relation between these two approaches, making their equivalence manifest (to all orders in the strong-coupling, i.e., $\frac{1}{\sqrt{\lambda}}$ expansion). In particular, we shall demonstrate that computing the quantum open $AdS_5 \times S^5$ string fluctuations near the cusp surface of [8] leads to the same 1-loop coefficient a_1 in (1.1) as found in the closed-string picture in [10].

The key observation [10,11] that simplifies dramatically the computation of quantum string corrections to the closed string energy in the large spin limit is that in order to compute the coefficient of the leading $\ln S$ term in E it is enough to consider a "scaling" limit of the full (elliptic function) solution of [7]. In this limit the string is stretched homogeneously along the radial direction of AdS_5 all the way to the boundary, i.e., $\rho = \kappa\sigma$, with $\kappa \approx \frac{1}{\pi}\ln\frac{S}{\sqrt{\lambda}} \gg 1$. One may also ignore the boundary (turning-point) contributions since they are subleading in the large $\frac{S}{\sqrt{\lambda}}$ limit. This scaling-limit solution is formally related [11,12] via an analytic continuation to the circular rotating string with two equal S^5 angular momenta $J_1 = J_2$ [18,19] and an imaginary value of

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[2] The expression for a_2 can be found in [12].