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S-matrix elements from T-duality

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Abstract

Recently it has been speculated that the S-matrix elements satisfy the Ward identity associated with the T-duality. This indicates that a group of S-matrix elements is invariant under the linear T-duality transformations on the external states. If one evaluates one component of such T-dual multiplet, then all other components may be found by the simple use of the linear T-duality. The assumption that fields must be independent of the Killing coordinate, however, may cause, in some cases, the T-dual multiplet not to be gauge invariant. In those cases, the S-matrix elements contain more than one T-dual multiplet which are intertwined by the gauge symmetry.

In this paper, we apply the T-dual Ward identity on the S-matrix element of one RR $(p-3)$-form and two NSNS states on the world volume of a D_p-brane to find its corresponding T-dual multiplet. In the case that the RR potential has two transverse indices, the T-dual multiplet is gauge invariant, however, in the case that it has one transverse index the multiplet is not gauge invariant. We find a new T-dual multiplet in this case by imposing the gauge symmetry. We show that the multiplets are reproduced by explicit calculation, and their low energy contact terms at order α'^2 are consistent with the existing couplings in the literature.

Keywords: T-duality; S-matrix

1. Introduction

It is known that the string theory is invariant under T-duality [1–5] and S-duality [5–11]. These symmetries should be carried by the scattering amplitudes. It has been speculated in [12] that these dualities at linear order should appear in the amplitudes through the associated Ward

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identities. This classifies the tree-level amplitudes into T-dual (S-dual) multiplets. Each multiplet includes the scattering amplitudes which interchange under the linear T-duality (S-duality) transformations.

The T-duality holds order by order in string loop expansion [5]. Hence, scattering amplitudes at any loop order should satisfy the T-dual Ward identity. This identity dictates that the amplitudes should be covariant under linear T-duality transformations on the external states and should be covariant under the full nonlinear T-duality transformations on the background fields. On the other hand, the S-duality is nonperturbative in string loop expansion [5]. The S-dual Ward identity then dictates the scattering amplitudes should be invariant under linear S-duality transformations on the external states [13–15] and should be invariant under the full nonlinear S-duality transformations on the background fields after including the loops and the nonperturbative effects [16–35].

In the T-duality transformations, there is an assumption that the background fields must be independent of the Killing coordinates along which the T-duality is applied [2]. As a result, the T-dual Ward identity cannot capture the T-dual multiplets that are proportional to the momentum along the killing coordinates. The gauge invariance associated with the massless closed string states, however, intertwine the separate T-dual multiplets. Therefore, one may find all T-dual multiplets by imposing the Ward identities associated with the T-duality and the gauge transformations.

The consistency of the standard Chern–Simons couplings of gravity and R–R fields [36,37] with the linear T-duality has been used in [38,39] as a guiding principle to find new couplings involving the antisymmetric B-field. The T-dual multiplet corresponding to the Chern–Simons couplings at order α'^2 has been found in [38,39]. It is not invariant under the B-field and the R–R gauge transformations. This reveals that the extension of the Chern–Simons couplings to the T-duality invariant form involves more than one T-dual multiplets. It has been shown in [39] that the multiplets may be found by imposing the invariance under these gauge transformations. Similar consideration should be applied on the scattering amplitudes corresponding to these couplings.

The disk-level scattering amplitude of one RR $(p-3)$-form and two NSNS states has been calculated in [40–42]. In this paper we would like to apply the T-dual Ward identity on this amplitude to find its corresponding T-dual multiplet. This multiplet contains the amplitudes for $(p-1)$-form, $(p+1)$-form, $(p+3)$-form, $(p+5)$-form as well as the original amplitude. As we shall see, in the case that the RR potential has two transverse indices the T-dual multiplet is invariant under the gauge transformations associated with the massless closed strings. However, in the case that the RR potential has one transverse index, the T-dual multiplet is not invariant under these gauge transformations. As a result, the S-matrix in this case involves two T-dual multiplets. We will find the second multiplet by imposing the Ward identity associated with the NSNS gauge transformations on the first multiplet.

The outline of the paper is as follows: We begin in Section 2 by reviewing the T-duality transformations and the method for finding the T-dual completion of an S-matrix element. In Section 3.1, we show that the S-matrix element of one RR $(p-3)$-form with two transverse indices and two NSNS states calculated in [40–42] does not satisfy the T-dual Ward identity. Imposing this identity, we will find the T-dual multiplet whose first component is the amplitude of the R–R $(p-3)$-form. This multiplet satisfies the Ward identities associated with the R–R and the NSNS gauge transformation. In Section 3.2, we find the T-dual multiplet whose first component is the amplitude of the RR $(p-3)$-form with one transverse index. This multiplet does not satisfy the Ward identity associated with the NSNS gauge transformations. In Section 3.3, by imposing