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Exactly solvable models in atomic and molecular physics

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Abstract

We construct integrable generalized models in a systematic way exploring different representations of the $gl(N)$ algebra. The models are then interpreted in the context of atomic and molecular physics, most of them related to different types of Bose–Einstein condensates. The spectrum of the models is given through the analytical Bethe ansatz method. We further extend these results to the case of the superalgebra $gl(M|N)$, providing in this way models which also include fermions.

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1. Introduction

Exactly solvable models are a fascinating issue that continue to attract considerable interest in physics and mathematics. Although the integrability of quantum systems is usually restricted to one dimension, there are many reasons that turn this study relevant for physical applications. It serves as a test for computer analysis and analytical methods for realistic systems to which, until now, only numerical calculations and perturbative methods may be applied. In addition, a non-trivial solvable model reveals the essence of the phenomena under consideration. For instance, many concepts established in critical phenomena were inspired by the exact solution of the Ising model. From the experimental point of view, there are some real spin-1 compounds (e.g. NENC, NDPK, or NBYC, etc.) [1] and strong coupling ladder compounds (such as $(5IAP)_2CuBr_4{\cdot}2H_2O$,

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or $Cu_2(C_5H_{12}N_2)_2Cl_4$, etc.) [2] that can be well described by integrable models. The necessity of using exactly solvable models has been also demonstrated through experimental research on aluminium grains at nanoscale level [3].

A significant aspect of integrable systems is its interdisciplinary character, i.e., they can be found in different areas of physics. The Ising and the Heisenberg models [4] in statistical mechanics, the t–J and Hubbard models [5] in condensed matter physics, the nonlinear σ-model [6] in quantum field theory, the interacting Boson model [7] in nuclear physics and more recently the two-site Bose–Hubbard model [8] in atomic and molecular physics are just some representative examples of the high impact and potentiality of these systems. Let us remark also the emergence of integrable systems in high energy physics, more particularly in gauge theories [9–11] (for a recent review, see [12]), or string theory, through the recent analysis in super-Yang–Mill theories, see e.g. [13,14].

Therefore, new exactly solvable models are highly welcome and constitute the main focus of the present article. In particular we will concentrate on the construction of integrable generalized models in atomic and molecular physics, most of them related to Bose–Einstein condensates (BECs).

The phenomenon of Bose–Einstein condensation, while predicted long ago [15], is currently one of the most active fields in physics, responsible for many of new perspectives on the potential applications of quantum systems. Since the early experimental realizations of BECs using ultracold dilute alkali gases [16], intense efforts have been devoted to the study of new properties of BEC. In recent years the creation of a molecular BEC from an atomic BEC has been obtained by different techniques [17]. The field was further broadened by the achievement of quantum degeneracy in ultracold fermionic gases [18]. These achievements could lead to new scientific investigations that includes coherent atomic lasers, quantum chemistry, the quantum gas with anisotropic dipolar interactions, quantum information, atomtronics, among many others.

In this context, it is natural to expect that exactly solvable models in the BEC scenario may be of relevance, providing some physical insights [19]. Our main purpose here is to employ the integrable systems machinery in its full power, i.e., exploring all possible types of representations of some algebra (we consider, in particular the $gl(N)$ algebra) to enlarge the family of known exactly solvable models in atomic and molecular physics with the aim that potentially new relevant models emerge. Using this machinery some existing models in the BEC scenario, such as the two-site Bose–Hubbard model [8] will be restored as well as new ones will be obtained. A two-coupled BEC model with a field, a two-coupled BEC-model with different types of atoms and a three-coupled BEC model, among others, will be introduced in this general framework. It is worth to mention here that the popular "BEC-transistor" in atomtronics uses a BEC in a triple well [20]. The models are then solved by means of Bethe ansatz methods.

Our paper is organized as follows: in Section 2 we briefly review the general setting of integrable systems and fix notation. In Section 3 we present our general approach and also discuss the different representations of the $gl(N)$ algebra that will be adopted. Section 4 is devoted to the discussion of the different physical models we can get using this construction. In Section 5 the Bethe ansatz equations of the models are derived. We extend these results to the case of the superalgebra $gl(M|N)$ in Section 6 and some applications of this formalism, i.e. models which also include fermions are presented in Section 7. Section 8 is devoted to some concluding remarks.