Available online at www.sciencedirect.com

SciVerse ScienceDirect

Nuclear Physics B 868 (2013) 16–37

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Loop variables and gauge invariant exact renormalization group equations for (open) string theory II

B. Sathiapalan

Institute of Mathematical Sciences, Taramani, Chennai, 600113, India

Received 17 September 2012; received in revised form 17 October 2012; accepted 5 November 2012

Available online 16 November 2012

Abstract

In arXiv:1202.4298 gauge invariant interacting equations were written down for the spin 2 and spin 3 massive modes using the exact renormalization group of a world sheet theory. This is generalized to all the higher levels in this paper. An interacting theory of an infinite tower of massive higher spins is obtained. They appear as a compactification of a massless theory in one higher dimension. The compactification and consequent mass is essential for writing the interaction terms. Just as for spin 2 and spin 3, the interactions are in terms of gauge invariant "field strengths" and the gauge transformations are the same as for the free theory. This theory can then be truncated in a gauge invariant way by removing one oscillator of the extra dimension to match the field content of BRST string (field) theory. The truncation has to be done level by level and results are given explicitly for level 4. At least up to level 5, the truncation can be done in a way that preserves the higher-dimensional structure. There is a relatively straightforward generalization of this construction to (arbitrary) curved space–time and this is also outlined.

Keywords: String theory; Renormalization group; String field theory

1. Introduction

In [1] (hereafter I) an exact renormalization group (ERG) [2–8] was written down for the world sheet theory describing a bosonic open string. The equations were worked out up to level 3 and had the following features:

E-mail address: bala@imsc.res.in.

0550-3213/$ – see front matter
http://dx.doi.org/10.1016/j.nuclphysb.2012.11.002

1. They are written in terms of loop variables, $k^\mu(t) = k_0^\mu + \frac{k_1^\mu}{t} + \cdots + \frac{k_n^\mu}{t^n} + \cdots$ and have the invariance $k^\mu(t) \to \lambda(t)k^\mu(t), \lambda(t) = 1 + \frac{\lambda_1}{t} + \frac{\lambda_n}{t^n} + \cdots$. When mapped to space–time fields, this maps to the gauge transformations of the space–time fields.
2. The equations are quadratic. This suggests that the interactions are cubic in the action, although we do not yet have an action.
3. The gauge transformations are the same as that of the free theory – *the interactions do not modify the form of the gauge transformations* unlike in Witten's BRST string field theory [9,10,12]. The interactions are written in terms of gauge invariant objects or "field strengths".[1]
4. The equations, at the free level, look exactly like those of a massless theory in one higher dimension. This idea has been widely used in the theory of higher spins [13–16]. The massive theory can be obtained by a compactification or some other kind of dimensional reduction, but at the level of the free theory compactification is optional. However the interactions can be written down in a gauge invariant manner only after dimensional reduction with mass. The gauge invariant field strength requires a mass parameter – which is the momentum in the internal direction. Thus $k^\mu(t), \mu = 1, \ldots, D+1$ becomes $k^\mu(t), q(t), \mu = 1, \ldots, D$. $q(t) = q_0 + \frac{q_1}{t} + \frac{q_2}{t^2} + \cdots + \frac{q_n}{t^n} + \cdots$. And q_0 is the mass. The gauge transformation of $q(t)$ is $q(t) \to \lambda(t)q(t)$.
5. It was shown in [10,11,9] that the auxiliary fields required for gauge invariance in BRST string field theory can be obtained as a subset of oscillator excitations of the ghost fields. In the bosonized ghost form this subset corresponds to setting to zero the first oscillator. In our case q_n are the counterparts of these oscillators and thus we need to get rid of q_1 in a consistent way.[2] Thus expressions containing q_1 have to be rewritten in terms of expressions that do not contain q_1 in such a way that the gauge transformations are preserved.
6. Even after the field content is matched with that of BRST string theory, the mass spectrum and dimension of the theory continue to be unconstrained by gauge invariance or other space–time symmetries. However when one requires that the gauge transformations and constraints match those of string theory, one recovers $D = 26$ and $q_0^2 = 2, 4$ for the first two massive levels, in addition to $q_0 = 0$ for the vector.
7. The ERG equations can be written down for any background and one does not have to perturb about a conformal background. At the free level gauge invariant equations for the massive spin 2 have been written down in arbitrary curved spaces using this method [18].[3] At the free level an action for the massive spin 2 in AdS space has also been written down [19].

In this paper we generalize the construction of the gauge invariant ERG to all levels. Thus we have gauge invariant equations of motion for an interacting theory of all spins in flat space–time. The only restriction is that they have to be massive. The equations continue to have the structure of a higher-dimensional theory dimensionally reduced with mass.

We then study the truncation to the set of fields describing BRST string field theory. This requires constructing a map from terms involving q_1 to terms without q_1 such that gauge invari-

[1] Some of these objects have the form of gauge invariant mass terms.

[2] A heuristic way to understand this is as follows: One can trade the $(D+1)$th coordinate for the Liouville mode σ [17], and then q_n is dual to $\frac{\partial^n \sigma}{\partial z^n}$. Since the first derivative of the metric can always be set to zero by a coordinate choice, $\frac{\partial \sigma}{\partial z}$ cannot correspond to any degree of freedom, and q_1 can therefore be removed.

[3] The technical complication involved is that the map from loop variables to space–time fields becomes more complicated and the curvature tensor of the background metric starts appearing.