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Asymptotic Bethe ansatz from string sigma model on $S^3 \times R$

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Abstract

We derive the asymptotic Bethe ansatz (AFS equations [G. Arutyunov, S. Frolov, M. Staudacher, Bethe ansatz for quantum strings, JHEP 0410 (2004) 016, hep-th/0406256]) for the string on $S^3 \times R$ sector of $AdS_5 \times S^5$ from the integrable nonhomogeneous dynamical spin chain for the string sigma model proposed in [N. Gromov, V. Kazakov, K. Sakai, P. Vieira, Strings as multi-particle states of quantum sigma-models, hep-th/0603043]. It is clear from the derivation that AFS equations can be viewed only as an effective model describing a certain regime of a more fundamental inhomogeneous spin chain.

1. Introduction

Recently, it was proposed [1] to describe the compact, $R \times S^5$ sector of the Green–Schwarz–Metsaev–Tseytlin (GSMT) superstring on $AdS_5 \times S^5$ background by the inhomogeneous, dynamical spin chain (we will abbreviate it to IDSC) built out of the physical particles of the quantum $SO(6)$ sigma model. The particles obey Zamolodchikovs factorizable S-matrix [2] and are put on the space circle representing the worldsheet direction of the closed string in the conformal

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gauge. Their isotopic $SO(6)$ vector degrees of freedom realize the target space projection to S^5. The proposal of [1] was inspired by a similar construction of [3], who considered a different, supersymmetric conformal $OSP(2m+2|2m)$ sigma-model.

This sigma model is asymptotically free and cannot pretend to the precise quantum description of the GSMT superstring. However, as was shown in [1], the Bethe ansatz equations of the model perfectly reproduce in the classical limit the finite gap solutions of [4,5] in terms of their algebraic curve.[3]

A natural question can be posed about the proposal of [1]: does it capture some essential features of the quantum superstring theory? For the moment we dispose a very limited information about the quantum behavior of GSMT superstring. The only robust calculations are made in the so-called BMN limit [6] where the quantum one loop $1/\sqrt{\lambda}$ [4] corrections were calculated [7], as well as the same corrections around the classical solutions for the simplest string motions: for the rotating circular and folded string [8–10].

In an interesting attempt to quantize the string on $R \times S^3$, the authors of [11] conjectured a discretization of the classical finite gap equations of [4] which in addition has the right BMN and the large gauge coupling limits. It was later pointed out [12,13] that the resulting equation for Bethe roots Eq. (20) (the so-called AFS equation) can capture only a part of the truth, having chances to describe only large λ and large angular momentum L of the string on S^3.

Nevertheless, the AFS equations appeared to be a useful empirical guideline for the search of quantum formalism based on integrability. First, they appear to be the Bethe ansatz equations of a certain non-local spin chain [14]. Second, they were generalized in a natural way to the full superstring theory in [15], following nice observations of [16]. This general model appears to be different from the asymptotic SYM Bethe ansatz Eq. (19), the so-called BDS equation, proposed in [17], the so-called BDS equation, only by a universal scalar factor σ^2 Eq. (21). The BDS is better justified and to great extent even deduced from the superalgebra of SYM theory [18,19]. The BDS equation is claimed to be constrained by the crossing relations [20]. It reproduces correctly at least three loops of anomalous dimensions of SYM theory [16]. Moreover, the quantum corrections of [8–10] were reproduced on the basis of AFS equations with some modifications in [13,21–23]. All this means that the empirical AFS equations contain some grain of truth about the structure of the quantum superstring.

The main problem with AFS equations is that they are not deduced from any general enough principle as an approximation with respect to a parameter, but rather proposed as an empirical guess.

In this paper, we will show that the AFS equation (43) follows for large λ and L from the integrable quantum inhomogeneous dynamical spin chain (IDSC) of [1] based on the sigma model on S^3. The IDSC plays similar role for the AFS equation as the Hubbard model [24] for the BDS equation of [17]. The derivation is quite straightforward and it demonstrates the qualitative nature of the AFS equations, explicitly containing large parameters L and λ. The IDSC model seems to be more simple and fundamental since it is a selfconsistent integrable quantum system. It may incorporate the known data for both perturbative small λ, SYM theory and the large λ quasiclassical string results since it has two a priory adjustable functions, dispersion of the

[3] In [3] it was done for the $OSP(2m+2|2m)$ sigma model, but the details of the classical limit are there quite different from ours.

[4] λ is the 't Hooft coupling constant of the dual $N=4$ SYM theory; $\hbar = 1/\sqrt{\lambda}$ plays the role of the Planck constant of the worldsheet sigma model.