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Roaming moduli space using dynamical triangulations

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Abstract

In critical as well as in non-critical string theory the partition function reduces to an integral over moduli space after integration over matter fields. For non-critical string theory this moduli integrand is known for genus one surfaces. The formalism of dynamical triangulations provides us with a regularization of non-critical string theory. We show how to assign in a simple and geometrical way a moduli parameter to each triangulation. After integrating over possible matter fields we can thus construct the moduli integrand. We show numerically for $c = 0$ and $c = -2$ non-critical strings that the moduli integrand converges to the known continuum expression when the number of triangles goes to infinity.

Keywords: Quantum gravity; Lower-dimensional models; Lattice models

1. Introduction

Non-critical string theory, or two-dimensional Euclidean quantum gravity coupled to matter was introduced by Polyakov [1], and has been a fruitful laboratory for studying aspects of string theory as well as quantum gravity. Solving these theories one has had the advantage to have both a lattice version of the theory and a continuum field theory formulation. The lattice version has been denoted the dynamical triangulation model (*DT*) or the matrix model of non-critical string theory [2–4]. It can be solved, basically by combinatorial methods, exemplified by the use of

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matrix models. The explicit solution of the continuum model uses the fact that the Liouville theory is a (special) conformal theory. For observables which can be calculated by both approaches agreement is found.

Since non-critical string theory is described by conformal field theory we know that conformal invariance is implemented. However, the precise manifestation of this invariance, and how it is related to the moduli space of the underlying surfaces (i.e. to the part of the surface geometry left invariant under conformal transformations) has been rather limited. That dynamical triangulations or matrix models contain precise information about the moduli space is on the other hand obvious. For instance it was shown in [5] that the resolvent of the matrix model has an expansion in terms of so-called moments, where the coefficients in the double scaling for the genus h terms are precisely the intersection indices of moduli space for genus h Riemann surfaces with any number of punctures. It was also shown how these matrix models in the double scaling limit can be related to the Kontsevich matrix model which provides a representation of the generating function for these intersection indices [5,6]. Much work has later expanded on these results [7], but we are in these approaches far from the naive and simple idea which was the starting point for *DT* and matrix models, namely that *DT* provides a regularization of the path integral of non-critical strings and thus the moduli parameters should appear in the same simple way as they formally appear in the string path integral.

One problem when using matrix models is that most observables which can be calculated analytically are of global nature: integrated correlation functions of matter fields or the partition function as a function of various boundary states. Further, the comparison with continuum results is only possible for the simplest topologies of the surfaces: spherical topology, disc topology and (only recently [8]) cylindrical topology. It is simply difficult to perform calculations in the framework of Liouville theory for higher genus surfaces. However, there is a narrow window where one can test in more detail if the matrix models, or the framework of dynamical triangulations, actually agree with the continuum expressions for higher genus surfaces in the naive way mentioned above. We know the partition function for genus one surfaces of non-critical strings expressed as an integral over the moduli parameter of the torus [9]. Integrating out the matter fields, using the conformal anomaly, worldsheet conformal invariance ensures that the remaining integrand of the partition function is only an (explicitly known) function of the moduli parameters. We can now in principle compare this integrand to the integrand which we obtain using dynamical triangulations as a regularization of the non-critical string theory. Which integrand *do* we obtain using dynamical triangulations? For each triangulation we have a continuous, piecewise linear geometry. To each such geometry we can associate moduli parameters (as described below). Each geometry will appear with a certain weight which depends on the matter we have coupled to the surface. If we have N triangles we will in this way obtain a number of *points* in moduli space and when N goes to infinity we expect these discrete points converge to a density distribution which should be proportional to the integrand of non-critical string theory. The possibility of performing such a comparison was pioneered by Kawai and collaborators [10]. They found good qualitative agreement between the non-critical string integrand and the density constructed from dynamical triangulations. The purpose of this article is to improve this test, making it quantitative, and also present a more general setup than the one used in [10]. We also show how one can measure the moduli parameters for higher genus triangulations, but unfortunately there is presently no theoretical calculation with which we could compare such results which would be easy to generate numerically.