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Classifying A-field and B-field configurations in the presence of D-branes. Part II: Stacks of D-branes

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Abstract

In the paper of Bonora et al. (2008) [3] we have shown, in the context of type II superstring theory, the classification of the allowed B-field and A-field configurations in the presence of anomaly-free D-branes, the mathematical framework being provided by the geometry of gerbes. Here we complete the discussion considering in detail the case of a stack of D-branes, carrying a non-abelian gauge theory, which was just sketched in Bonora et al. (2008) [3]. In this case we have to mix the geometry of abelian gerbes, describing the B-field, with the one of higher-rank bundles, ordinary or twisted. We describe in detail the various cases that arise according to such a classification, as we did for a single D-brane, showing under which hypotheses the A-field turns out to be a connection on a canonical gauge bundle. We also generalize to the non-abelian setting the discussion about "gauge bundles with non-integral Chern classes", relating them to twisted bundles with connection. Finally, we analyze the geometrical nature of the Wilson loop for each kind of gauge theory on a D-brane or stack of D-branes.

Keywords: Algebraic topology; D-branes

1. Introduction

In order to describe a type II superstring background with a non-trivial B-field, a suitable mathematical tool is the geometry of gerbes with connection. There are many different approaches to this topic, but the most natural one in physics consists of using the Čech–Deligne hypercohomology of sheaves. The hypercohomology group of degree 1 describes abelian gauge theories, where the local potentials are 1-forms A_μ and the field strength is a gauge-invariant

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2-form $F_{\mu\nu}$, while the group of degree 2 describes the possible B-field configurations, where the local potentials are 2-forms $B_{\mu\nu}$ and the field strength is a gauge-invariant 3-form $H_{\mu\nu\rho}$.[1] When D-branes are present, the B-field and the A-field are not independent one from each other in general, but there is an interaction between them, which is needed in order for the world-sheet path-integral to be well defined: this kind of interaction is not possible for every D-brane world-volume, and the obstruction for it to exist is the Freed–Witten anomaly [6]. Therefore, a joint classification of the allowed A-field and B-field configurations is needed, and it can be reached via a certain hypercohomology group, or via a coset of it within a bigger group, as we discussed in [3]. From this picture it follows that, while the B-field is always a connection on a gerbe, the A-field is not always a connection on an ordinary $U(1)$ gauge bundle on the D-brane, even if this is the most common situation. In fact, there are different possibilities arising from this classification scheme, and only under suitable hypotheses we recover an abelian gauge theory in the usual sense. Actually, even in this case it is possible that there exists a residual gauge freedom, depending on the topology of the background.

When we deal with a stack of D-branes, usually carrying a $U(n)$ gauge theory, the previous classification scheme needs to be generalized. Something new must appear, since even the formulation of the Freed–Witten anomaly changes [11], because of the presence of a torsion cohomology class which is always vanishing in the abelian case. The idea leading to the classification is the same, but we need to deal with the degree 1 non-abelian cohomology [5], describing $U(n)$ bundles, and the degree 1 hypercohomology, describing $U(n)$ bundles with connection; contrary to the abelian case, we do not obtain a group but a pointed set, the marked point being the trivial bundle for cohomology and the trivial bundle with trivial connection for hypercohomology. The B-field, instead, remains abelian as always. Therefore, when the A-field and the B-field interact in order to make the world-sheet path-integral well defined, we must take into account this difference in their geometrical nature, especially when the A-field is not an ordinary connection. The main consequence of this new picture is that, while in the abelian case the A-field acts only as a gauge transformation of the B-field, without changing its geometry, in the non-abelian case it is possible that its presence carries a non-trivial geometry even with respect to the degree 2 hypercohomology (which classifies the B-field). Therefore, instead of acting as a gauge transformation, it acts as a tensor product by a gerbe which is non-trivial in general, and this is the origin of the new term in the Freed–Witten anomaly. We thus need to give a careful description of this different action of the A-field, arriving in this way to the new classification scheme and its underlying geometry.

There are important physical consequences of all this. We will see that, for every D-brane world-volume such that the B-field gerbe, restricted to it, has a torsion first Chern class $[H]$ (that happens when the H-flux is exact on the world-volume as a differential form), it is always possible to find a gauge bundle such that the Freed–Witten anomaly vanishes, thanks to the term appearing only in the non-abelian case. Therefore, if we allow stacks of D-branes, the only condition (still strong!) that the Freed–Witten anomaly imposes on the world-volume is that $[H]$ is torsion; then, in order for the anomaly to vanish, in some cases it is necessary that the rank of the gauge bundle is sufficiently high, but this is a condition on the gauge theory, not on the world-volume. In particular, if H is exact on the whole space–time, there are no Freed–Witten anomalous world-volumes, even if some of them have constraints on the rank of the gauge theory.

[1] Similarly, the hypercohomology group of degree p describes the configurations of the Ramond–Ramond field whose local potentials are the p-forms $C_{\mu_1 \cdots \mu_p}$ and whose field strength is the gauge-invariant $(p+1)$-form $G_{\mu_1 \cdots \mu_{p+1}}$.