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Nuclear Physics B 909 (2016) 73–85

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Exact asymmetric Skyrmion in anisotropic ferromagnet and its helimagnetic application

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Received 12 October 2015; received in revised form 13 April 2016; accepted 27 April 2016

Available online 4 May 2016

Editor: Hubert Saleur

Abstract

Topological Skyrmions as intricate spin textures were observed experimentally in helimagnets on 2d plane. Theoretical foundation of such solitonic states to appear in pure ferromagnetic model, as exact solutions expressed through any analytic function, was made long ago by Belavin and Polyakov (BP). We propose an innovative generalization of the BP solution for an anisotropic ferromagnet, based on a physically motivated geometric (in-)equality, which takes the exact Skyrmion to a new class of functions beyond analyticity. The possibility of stabilizing such metastable states in helimagnets is discussed with the construction of individual Skyrmion, Skyrmion crystal and lattice with asymmetry, likely to be detected in precision experiments.

1. Introduction

Unusual spin textures, identified as magnetic Skyrmions of topological origin, were observed in helimagnetic crystals like Mn Si and $Fe_xCo_{1-x}Si$, in a series of recent experiments [1–3]. However, the possibility of such a phenomenon to occur on a two-dimensional (2d) plane in ferromagnetic model was predicted by Belavin and Polyakov 40 years back, in a pioneering work, where they found exact Skyrmions in a general framework with topological charge $Q = N$, linked to any analytic functions [4]. Problem with this beautiful theoretical result was, that due

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http://dx.doi.org/10.1016/j.nuclphysb.2016.04.043

to scale invariance of the solution it could give only metastable states, where for stabilizing the Skyrmions one needs to bring in coupling parameters through additional interactions. In a helimagnetic model a Dzyaloshinskii–Moriya (DM) spin-orbital interaction term is added to the spin–spin ferromagnetic interaction, where the DM coupling breaks the unwanted scale invariance and stabilizes the solitons through competing forces between the ferromagnetic and the effective DM interactions. Recent experimental observation of Skyrmions in magnetic models is therefore a landmark confirmation of an old basic theoretical result proposed in [4].

It is a bit surprising, that in the recent large collection of high-profile theoretical and experimental work dedicated to the magnetic Skyrmions in helimagnets [1–3,5,6], rarely the work of BP [4] is mentioned, which on the other hand is the theoretical basis for the topological Skyrmion in magnetic model in 2d. It is also rather unexpected, that no other extension or generalization of the BP solutions was proposed in 2d, over these long years of development in the subject. Our motivation here, is to bring in a novel contribution to the subject, by discovering a family of exact Skyrmions for an anisotropic extension of the ferromagnetic model, through a physically motivated geometric (in-)equality, which generalizes the BP result by going beyond the analytic functions and linking the Skyrmions to the contemporary $\bar{\partial}$ problem [8]. Such Skyrmions exhibit more inherent asymmetry and anisotropy, properties akin also to the helimagnetic crystals with noninversion symmetry and anisotropy exhibited in 2d. Therefore, a natural application of such asymmetric solitons to the helimagnetic model, through their stabilization under DM interaction might be a promising perspective to be verified in a precision experiment.

2. Ferromagnetic and helimagnetic models and BP solution

As evident from the experimental images, the Skyrmion magnetic pattern in helimagnets shows extended structure for individual Skyrmions in a 30 nm range, slowly varying over several lattice spacings [3]. Therefore, one can go for the continuum limit reducing the ferromagnetic Hamiltonian H_f from the Heisenberg spin model on a 2d lattice with nearest neighbor interactions as

$$H_{HS} = \frac{1}{2} \sum_{<\mathbf{j},\mathbf{j}'>} \mathbf{s_j s_{j'}} \longrightarrow \quad H_f = \frac{1}{2} \int d^2x (\nabla \mathbf{M})^2, \tag{1}$$

where $\mathbf{s_j} \rightarrow \mathbf{M}$, $\mathbf{s_j^2} \rightarrow \mathbf{M}^2 = 1$. For constructing a helimagnetic model a crucial addition of a spin-orbital interaction is needed to the ferromagnetic Hamiltonian (1) in the form of a DM term

$$H_{dm} = \int d^2x (\mathbf{M} \cdot [\nabla \times \mathbf{M}]), \tag{2}$$

which was proposed phenomenologically way back in another landmark work [7].

Note, that the DM Hamiltonian (2) exhibits an explicitly broken space inversion symmetry as well as anisotropy, due to the appearance of only first order derivatives and interaction of the 2d coordinate space with the internal spin space, rewriting (2) as

$$H_{dm} = \int d^2x \left(M^1 \partial_y M^3 + M^2 \partial_x M^3 + M^3 (\partial_x M^2 - \partial_y M^1) \right). \tag{3}$$

Therefore the basic helimagnetic model, for investigating the magnetic Skyrmions of contemporary interest, may be given simply by the model Hamiltonian $H_{heli} = J H_f + D\, H_{dm}$ where J is the spin exchange parameter, D is the spin-orbital coupling and the ferromagnetic H_f and the DM Hamiltonian H_{dm} may be given by (1) and (3), respectively.