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Quantum correlations in terms of neutrino oscillation probabilities

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Abstract

Neutrino oscillations provide evidence for the mode entanglement of neutrino mass eigenstates in a given flavour eigenstate. Given this mode entanglement, it is pertinent to consider the relation between the oscillation probabilities and other quantum correlations. In this work, we show that all the well-known quantum correlations, such as the Bell's inequality, are directly related to the neutrino oscillation probabilities. The results of the neutrino oscillation experiments, which measure the neutrino survival probability to be less than unity, imply Bell's inequality violation.

1. Introduction

The foundations of quantum mechanics are usually studied in optical or electronic systems. In such systems, the interplay between the various measures of quantum correlations is well known. Inspired by the recent technical advances in high energy physics experiments, in particular the meson factories and the long baseline neutrino experiments, attention has also been directed towards subatomic physics [1–10].

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The study of quantum correlations in neutrinos, has been mostly focussed on entanglement [3–6]. Recently, a temporal analogue of Bell's inequality, the Leggett–Garg inequality, has been studied in the context of neutrino oscillations [10]. Here, along with entanglement we also study the other quantum correlations such as Bell's inequality violation, teleportation fidelity and geometric discord in the context of two flavour neutrino oscillations as well as study the interplay between them. In particular, we show that all these quantities are directly related to the neutrino oscillation probabilities.

Neutrino oscillations are experimentally well established [11–17]. Such oscillations are possible if both of the following conditions are satisfied:

- The neutrino flavour state is a linear superposition of non-degenerate mass eigenstates.
- The time evolution of a flavour state is a coherent superposition of the time evolution of the corresponding mass eigenstates.

The coherent time evolution implies that there is *mode entanglement* between the mass eigenstates which make up a flavour state. Such mode entangled states have been the subject of intense discussions over the last two decades [18–23], resulting in the general consensus of subspace entanglement as a generalized feature of inter particle entanglement [23]. It has been the subject of many theoretical and experimental proposals [21] as well as successful experimental realizations [22] in atom-photon systems. Here we use the concept of mode entanglement to relate flavour oscillations to bipartite entanglement of single particle states.

The quest for understanding quantum correlations could be thought to have begun with the efforts of Einstein–Podolsky–Rosen (EPR) [24]. A quantitative understanding of EPR led to the development of Bell's inequality [25], with refinements leading to the Bell-CHSH (Clauser–Horn–Shimony–Holt) inequalities [26]. Violation of Bell's inequality quantifies the non-locality inherent in the system. A weaker, though very popular and widely studied facet of quantum correlations, is entanglement [27]. This has been applied to understand the process of teleportation [28]. A still weaker measure is quantum discord [29,30] and was developed as the difference between the quantum generalizations of two classically equivalent formulations of mutual information. States which are separable and hence have no entanglement could still have non-zero discord. Hence, our present understanding of quantum correlations is that it is a complex entity with many facets. There is now an abundance of measures of quantum correlations such as quantum work deficit [31], measurement induced disturbance [32] and dissonance [33].

In this paper we study a number of quantum correlations in the context of two-flavour neutrino oscillations. Among them are mode non-locality, concurrence, discord and teleportation fidelity. We find that all these quantum correlations are simple functions of the neutrino oscillation probabilities. A non-zero oscillation probability immediately leads to a violation of Bell's inequality and to a teleportation fidelity value of greater than 2/3.

We first provide an introduction to the quantum mechanics of two flavour neutrino oscillations. Here we see that mode entanglement comes in a natural setting. We then discuss and compute different quantum correlations and relate them to the neutrino oscillation probabilities. We finish with our conclusions.

2. Quantum mechanics of two flavour neutrino oscillations

It is well known that there are three flavour states of neutrinos, ν_e, ν_μ and ν_τ [34,35]. In the oscillation formalism, it is assumed that they mix via a 3×3 unitary matrix to form the three mass