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T-duality of α'-correction to DBI action at all orders of gauge field

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Abstract

By explicit calculations of four-field couplings, we observe that the higher derivative corrections to the DBI action in flat space–time, can be either in a covariant form or in a T-duality invariant form. The two forms are related by a non-covariant field redefinition. Using this observation, we then propose a non-covariant but T-duality invariant action which includes all orders of massless fields and has two extra derivatives with respect to the DBI action.

1. Introduction

The effective action of a D_p-brane in bosonic string theory includes various world-volume couplings of open string tachyon, transverse scalar fields, gauge field, closed string tachyon, graviton, dilaton and B-field. Because of the tachyons, the bosonic string theory and its D_p-branes are all unstable. Assuming the tachyons are frozen at the top of their corresponding tachyon potentials, the effective action at the leading order of α' in flat spacetime is then given by Dirac–Born–Infeld (DBI) action [1,2]

$$S_p \supset -T_p \int d^{p+1}\sigma \sqrt{-\det(\widetilde{G}_{ab} + F_{ab})} \tag{1}$$

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where $\widetilde{G}_{ab}$ is pull-back of the bulk flat metric onto the world-volume,[1] *i.e.*,

$$\begin{aligned}\widetilde{G}_{ab} &= P[\eta]_{ab} = \frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}\eta_{\mu\nu}\\ &= \eta_{ab} + \partial_a\chi^i\partial_b\chi^j\eta_{ij}\end{aligned} \tag{2}$$

where in the second line the pull-back is in static gauge,[2] *i.e.*, $X^i = \chi^i$ and $X^a = \sigma^a$. The DBI action (1) is covariant under the general coordinate transformations and is invariant under T-duality.[3] With our normalization for the gauge field, the DBI action is at the leading order of α'. The first correction to this action is at order α' in which we are interested in this paper. The α' corrections to Born–Infeld action which includes only gauge field, have been studied in [3–8]. Using the proposed connection between relativistic hydrodynamics and open string effective action [9,10], the α' corrections to the DBI action may be used for studying higher-derivative corrections in relativistic hydrodynamics.

At zero gauge field level, the general covariance requires the world-volume couplings at any order of α' consist of various contractions of the second fundamental form $\Omega_{ab}{}^\mu = D_a\frac{\partial X^\mu}{\partial\sigma^b}$ and its covariant derivatives. By studying the disk-level S-matrix element of two graviton vertex operators at low energy, such couplings at order α' have been found in [11] to be

$$\begin{aligned}S_p \supset -T_p\int d^{p+1}\sigma\sqrt{-\det(\widetilde{G}_{ab})}\Big[1+\frac{\alpha'}{2}R\\ +\alpha'\widetilde{\perp}_{\mu\nu}\widetilde{G}^{ab}\widetilde{G}^{cd}\Big(\Omega_{ab}{}^\mu\Omega_{cd}{}^\nu-\Omega_{ac}{}^\mu\Omega_{bd}{}^\nu\Big)\Big]\end{aligned} \tag{3}$$

where $R = \widetilde{G}^{\mu\nu}\widetilde{G}^{\alpha\beta}R_{\mu\alpha\nu\beta}$ and $\widetilde{G}^{\mu\nu}$ is the first fundamental form, *i.e.*,

$$\widetilde{G}^{\mu\nu} = \frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}\widetilde{G}^{ab}, \tag{4}$$

which is a projection operator, *i.e.*, $\eta_{\nu\alpha}\widetilde{G}^{\mu\nu}\widetilde{G}^{\alpha\beta} = \widetilde{G}^{\mu\beta}$. It projects space–time tensors to the world-volume space. The tensor $\widetilde{\perp}_{\mu\nu}$ in (3) is a projection operator, *i.e.*, $\eta^{\nu\alpha}\widetilde{\perp}_{\mu\nu}\widetilde{\perp}_{\alpha\beta} = \widetilde{\perp}_{\mu\beta}$, which projects space–time tensors to the transverse space. This projection operator can be written in terms of the first fundamental form as

$$\widetilde{\perp}_{\mu\nu} = \eta_{\mu\nu} - \widetilde{G}_{\mu\nu} \tag{5}$$

In flat spacetime and in the static gauge, the transverse component of the second fundamental form in (3) is

[1] Our index convention is that the Greek letters $(\mu,\nu,\cdots)$ are the indices of the space–time coordinates, the Latin letters $(a,d,c,\cdots)$ are the world-volume indices and the letters $(i,j,k,\cdots)$ are the normal bundle indices. The killing index in the reduction of 10-dimensional space–time to 9-dimensional space–time is y.

[2] In the literature, there is a factor of $2\pi\alpha'$ in front of gauge field strength F_{ab} in the DBI action. We normalize the gauge field to absorb this factor. With this normalization, the gauge field along the killing direction, A_y, transforms to the transverse scalar field χ^y under T-duality.

[3] By invariance under T-duality, we mean after expanding the action to a specific order of field, the couplings at each order satisfy the T-duality constraint (26).