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Heterotic reduction of Courant algebroid connections and Einstein–Hilbert actions

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Dedicated to Satoshi Watamura on the occasion of his 60th birthday

Abstract

We discuss Levi-Civita connections on Courant algebroids. We define an appropriate generalization of the curvature tensor and compute the corresponding scalar curvatures in the exact and heterotic case, leading to generalized (bosonic) Einstein–Hilbert type of actions known from supergravity. In particular, we carefully analyze the process of the reduction for the generalized metric, connection, curvature tensor and the scalar curvature.

1. Introduction

This paper contains a thorough discussion of Riemannian geometry on exact and heterotic Courant algebroids, i.e., within the framework of generalized geometry as introduced by Hitchin [20] and further developed in [16,18,19]. The discussion here is in some aspects analogous to the Kaluza–Klein (KK) theory; See [4] for a nice review of KK. In the KK theory one starts with a metric on a principal G-bundle P, with a (compact) Lie group G. A G-invariant metric on P

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determines (and is determined by) a metric on the base manifold M, a principal connection on P and a G-invariant metric on each fiber G_x, smoothly depending on the base point x. Let us recall, to P there is the associated Atiyah algebroid L and the connection A corresponds to a choice of splitting of the corresponding Atiyah sequence. One can use the Levi-Civita connection on P and compute the Ricci scalar. The resulting Einstein–Hilbert type of action contains among others the ordinary Einstein–Hilbert action with the pure Yang–Mills theory. Also, let us recall that the KK-reduction naturally incorporates the dilaton.

Here, we modify this in the spirit of the generalized geometry. We can start with the generalized cotangent bundle $TP \oplus T^*P$ equipped with the structure of an exact Courant algebroid. If the principal action is the so called trivially extended one (and the first Pontryagin class of P vanishes), the exact Courant algebroid structure on P can be reduced to a Courant algebroid structure. In case of a Lie group G, whose Lie algebra $\mathfrak{g}$ is equipped with an ad-invariant non-degenerate bilinear form $\langle \cdot, \cdot \rangle_{\mathfrak{g}}$ (e.g. compact, semisimple), the resulting Courant algebroid E' is not an exact one, it is a so called heterotic Courant algebroid, its underlying vector bundle is the Whitney sum of the Atiyah algebroid L and the cotangent bundle T^*M. Vice versa, each such a heterotic Courant algebroid comes as a reduction from an exact Courant algebroid on P [5]. Similar statements can be made with respect to the respective generalized metrics. In this paper we thoroughly investigate the reduction on the level of Levi-Civita connections and the corresponding generalized Einstein–Hilbert actions. Roughly speaking, starting with an exact Courant algebroid (with the Dorfman bracket twisted by a closed 3-form H), equipped with a generalized metric (g, B) we arrive (ignoring the dilaton) at the generalized scalar curvature

$$\mathcal{R} = \mathcal{R}(g) - \frac{1}{12} H'_{klm} H'^{klm}$$

with $H' = H + dB$. Similarly, starting with a heterotic Courant algebroid, we arrive (again in the simplest case and ignoring the dilaton and the cosmological constant) at

$$\mathcal{R} = \mathcal{R}(g) - \frac{1}{12} H'_{klm} H'^{klm} + \frac{1}{4} \langle F'_{kl}, F'^{kl} \rangle_{\mathfrak{g}},$$

where F' is the curvature of a connection which is the sum of the starting principal connection A on P and an adjoint bundle valued one form A' on M entering the parametrization of a generalized metric (g, B, A') on a heterotic Courant algebroid, and $H' = H + dB + \ldots$. In this paper, among other things, we describe in detail, how the two above actions can be related by the reduction with respect to the trivially extend action of G. This relation will appear to be less straightforward as it might seem at the first glance. Let us note that the constants $-1/12$ and $1/4$ are related to the choice of normalizations of the fields (H, B, A, A') as these appear naturally from the generalized geometry of Courant algebroids. E.g., B is exactly the one entering the sum $g + B$.

The relevance of the heterotic Courant algebroids is due to the condition of the triviality of the first Pontryagin class. As noted, e.g., in [15,5], it is exactly the Green–Schwarz anomaly cancellation condition when the principal bundle P is a fiber product of a Yang–Mills bundle and the (oriented) frame bundle on M. Hence, the structure of a heterotic Courant algebroids can be used to naturally incorporate the corresponding α' correction. Related to this, recently, the heterotic effective actions, Green–Schwarz mechanism and the related α' correction have been extensively examined within the double field theory [6,25–27,34].[1] It would be interesting

[1] For a general review of double field theory, including discussion of effective action see [23,1].