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# Black hole with quantum potential

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**Abstract**

In this work, we investigate black hole (BH) physics in the context of quantum corrections. These quantum corrections were introduced recently by replacing classical geodesics with quantal (Bohmian) trajectories and hence form a quantum Raychaudhuri equation (QRE). From the QRE, we derive a modified Schwarzschild metric, and use that metric to investigate BH singularity and thermodynamics. We find that these quantum corrections change the picture of Hawking radiation greatly when the size of BH approaches the Planck scale. They prevent the BH from total evaporation, predicting the existence of a quantum BH remnant, which may introduce a possible resolution for the catastrophic behavior of Hawking radiation as the BH mass approaches zero. Those corrections also turn the spacelike singularity of the black hole to be timelike, and hence this may ameliorate the information loss problem.


## 1. Introduction

Recently, a new semi-classical approach for quantum gravity has been suggested in [1], in which it was shown that replacing classical trajectories, or geodesics, by quantal (Bohmian) trajectories leads to corrections to the Raychaudhuri equation. Hence, these new quantum corrections will affect all reasonable spacetimes which are incomplete or singular in a certain sense depending on the validity of the classical Raychaudhuri equation according to Hawking–Penrose

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theorem [2]. The quantum Raychaudhuri equation (QRE) has been found to prevent focusing of geodesics, and hence prevents the formation of singularities [1]. This has been investigated in cosmology with Friedmann–Robertson–Walker (FRW) Universe and it was found that the big bang singularity may be resolved using quantal geodesics [3]. It was also found that the Friedmann equation receives a quantum correction term that could be interpreted as a cosmological constant that gives a correct estimate of its observed value [3,4].

The QRE has been derived in [1] by considering a quantum mechanical description of a fluid or condensate. This condensate is described by a wavefunction $\psi = \mathcal{R}e^{iS}$, which is assumed to be normalizable and single valued, $\mathcal{R}(x^a)$ and $S(x^a)$ are real functions associated with the four velocity field $u_a = (\hbar/m)\partial_a S$. The expansion scalar is given by $\theta = Tr(u_{a;b}) = h^{ab}u_{a;b}$, where the transverse metric $h_{ab} = g_{ab} + u_a u_b$. The quantum Raychaudhuri equation for timelike geodesics, with vanishing shear and twist for simplicity, takes the form after some derivations [1,3]

$$\frac{d\theta}{d\lambda} = -\frac{1}{3}\,\theta^2 - R_{ab}u^a u^b + \frac{\hbar^2}{m^2}h^{ab}\left(\frac{\Box\mathcal{R}}{\mathcal{R}}\right)_{;a;b} + \frac{\epsilon_1\hbar^2}{m^2}h^{ab}R_{;a;b}\,. \tag{1}$$

$R_{a,b}$ and $R$ are the Ricci tensor and Ricci scalar respectively. The constant $\epsilon_1 = 1/6$ for conformally invariant scalar fluid, but left arbitrary here.

Since the black hole is an ideal laboratory to investigate quantum gravity, in this paper, we derive the quantum Raychaudhuri equation for null geodesics, and use it to derive a modified Schwarzschild metric. We then derive the quantum corrected thermodynamics of the black hole. We also investigate the impact of quantum corrections on the physical singularity of the black hole.

## 2. Quantum Raychaudhuri equation for null geodesics

In the case of null geodesics, the classical Raychaudhuri equation with vanishing shear and twist takes the form [5, p. 50]

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - R_{ab}k^a k^b, \tag{2}$$

where $k^a$ is a null tangent vector field and the expansion parameter $\theta$ is given by

$$\theta = k^a_{\ ;a}. \tag{3}$$

The transverse metric $h_{ab}$ is given by [5, p. 46]

$$h_{ab} = g_{ab} + k_a N_b + N_a k_b, \tag{4}$$

where $N_a$ is an auxiliary null vector field such that $k^a N_a = -1$ and $N^a N_a = 0$. To derive the quantum Raychaudhuri equation for null geodesics, we start with a Klein–Gordon-type equation with $m = 0$

$$\left(\Box - \epsilon_1 R - \epsilon_2 \frac{i}{2} f_{cd}\sigma^{cd}\right)\Phi = 0, \tag{5}$$

where $R$ is the curvature scalar, and $\epsilon_1 = 1/6$ for conformally invariant scalar field. The 4-momentum and “coordinate velocity” are defined as [6–8]

$$p_a = \hbar\partial_a S, \tag{6}$$

$$\vec{v} = \frac{d\vec{x}}{dt} = -c^2\frac{\vec{\nabla}S}{\partial^0 S}, \tag{7}$$