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Generalization of the Randall–Sundrum solution

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Abstract

The generalization of the Randall–Sundrum solution for the warp factor $\exp[\sigma(y)]$ in the scenario with one extra coordinate y, non-factorizable space–time geometry and two branes is obtained. It is shown that the function obtained $\sigma(y)$ is symmetric with respect to an interchange of two branes. It also obeys the orbifold symmetry $y \to -y$ and explicitly reproduces jumps of its derivative on both branes. This solution is defined by the Einstein–Hilbert's equations up to a constant C. It is demonstrated that different values of C result in theories with quite different spectra of the Kaluza–Klein gravitons.

1. Introduction

The 5-dimensional space–time with non-factorizable geometry and two branes was suggested by Randall and Sundrum (RS1 model) [1] as an alternative to the ADD model with flat extra dimensions [2–4]. Its phenomenological implications were explored soon [5]. The model predicts an existence of heavy Kaluza–Klein excitations (KK gravitons). These massive resonances are intensively searched for by the LHC Collaborations (see, for instance, [6,7]).

The RS scenario is described by the following background warped metric

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} \, dx^\mu \, dx^\nu - dy^2 \, , \tag{1}$$

where $\eta_{\mu\nu}$ is the Minkowski tensor with the signature $(+,-,-,-)$, and y is an extra coordinate. It is a model of gravity in the AdS_5 space–time compactified to the orbifold S^1/Z_2. There are

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two branes located at the fixed points of the orbifold. The function $\sigma(y)$ in the warp factor $\exp[-2\sigma(y)]$ was obtained to be [1]

$$\sigma_{\mathrm{RS}}(y) = \kappa|y| \,, \tag{2}$$

where κ is a parameter with a dimension of mass.

This expression is consistent with the orbifold symmetry $y \to -y$. However, it is not symmetric with respect to the branes. The jump of the derivative $\sigma'(y)$ on the brane $y = \pi r_c$ does not follow from expression (2) *directly*, but only after taking into account periodicity condition.[1] Moreover, a constant can be safely added to $\sigma(y)$. Thus, a generalization of the RS solution (2) is needed.

In the present paper we will derive such a general solution $\sigma(y)$ of the Einstein–Hilbert's equations which has the following properties: (i) it has the orbifold symmetry $y \to -y$; (ii) jumps of $\sigma'(y)$ are explicitly reproduced on both branes; (iii) it is symmetric with respect to the interchange of the branes; (iv) it includes a constant term.

Previously, the solution for $\sigma(y)$ was studied in ref. [8]. In the present paper we reconsider and strengthen arguments used in deriving this solution, as well as correct expressions for $\sigma'(y)$ and 5-dimensional cosmological constant Λ presented in [8]. Moreover, the solution in [8] was incomplete, since it did not contain an additional dimensionless quantity C ($0 \leqslant C \leqslant |\kappa|\pi r_c$). As it is shown in the present paper, a physical content of a theory depends crucially on a particular value of C.

In Section 2 a generalization of the Randall–Sundrum solution (2) is derived, and in Section 3 properties of a new solution are discussed in detail.

2. RS solution and its generalization

The classical action of the Randall–Sundrum scenario [1] is given by

$$\begin{aligned} S = &\int d^4x \int_{-\pi r_c}^{\pi r_c} dy \sqrt{G}\,(2\bar{M}_5^3 \mathcal{R} - \Lambda) \\ &+ \int d^4x \sqrt{|g^{(1)}|}\,(\mathcal{L}_1 - \Lambda_1) + \int d^4x \sqrt{|g^{(2)}|}\,(\mathcal{L}_2 - \Lambda_2) \,, \end{aligned} \tag{3}$$

where $G_{MN}(x, y)$ is the 5-dimensional metric, with $M, N = 0, 1, 2, 3, 4$, $\mu = 0, 1, 2, 3$, and y is the 5-th dimension coordinate of the size πr_c. The quantities

$$g^{(1)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = 0) \,, \quad g^{(2)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = \pi r_c) \tag{4}$$

are induced metrics on the branes, $\mathcal{L}_1$ and $\mathcal{L}_2$ are brane Lagrangians, $G = \det(G_{MN})$, $g^{(i)} = \det(g^{(i)}_{\mu\nu})$.

The periodicity condition, $y = y \pm 2\pi r_c$, is imposed and the points (x_μ, y) and $(x_\mu, -y)$ are identified. So, one gets the orbifold S^1/Z_2. We consider the case with two 3-branes located at the fixed points $y = 0$ (Plank brane) and $y = \pi r_c$ (TeV brane). The SM fields are constrained to the TeV (physical) brane, while the gravity propagates in all spatial dimensions.

[1] Here and in what follows, the *prime* denotes the derivative with respect to variable y.