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Quasi-local conserved charges of spin-3 topologically massive gravity

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Abstract

In this paper we obtain conserved charges of spin-3 topologically massive gravity by using a quasi-local formalism. We find a general formula to calculate conserved charge of the spin-3 topologically massive gravity which corresponds to a Killing vector field ξ. We show that this general formula reduces to the previous one for the ordinary spin-3 gravity presented in [18] when we take into account only transformation under diffeomorphism, without considering generalized Lorentz gauge transformation (i.e. $\lambda_\xi = 0$), and by taking $\frac{1}{\mu} \to 0$. Then we obtain a general formula for the entropy of black hole solutions of the spin-3 topologically massive gravity. Finally we apply our formalism to calculate energy, angular momentum and entropy of a special black hole solution and we find that obtained results are consistent with previous results in the limiting cases. Moreover our results for energy, angular momentum and entropy are consistent with the first law of black hole mechanics.

1. Introduction

Higher spin gravity was formulated by Vasiliev and collaborators in papers [1]. In its simplest form it is an extension of ordinary gravity that includes a massless scalar and massless fields with spins $S = 3, 4, \ldots$. In [2] Vasiliev proposed a system of gauge invariant nonlinear

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dynamical equations for totally symmetric massless fields of all spins in (A)dS backgrounds. According to the result of this paper, "in the framework of gravity, unbroken higher spin gauge symmetries require a non-zero cosmological constant". In paper [3] the authors have considered the coupling of a symmetric spin-3 gauge field $\varphi_{\mu\nu\lambda}$ to 3-dimensional gravity in a second order metric-like formulation. In the context of frame-like approach the gravitational coupling of a symmetric tensor of rank 3 in the presence of negative cosmological constant can be given by $SL(3,\mathbb{R}) \times SL(3,\mathbb{R})$ Chern–Simons theory [4–6]. The emergence of W-algebras as asymptotic symmetries of higher-spin gauge theories coupled to three-dimensional Einstein gravity with a negative cosmological constant has been discussed in [4].

Higher-spin theories in AdS_3, like ordinary gravity, possess no propagating degrees of freedom [4,7]. Pure Einstein–Hilbert gravity in three dimensions exhibits no propagating physical degrees of freedom [8,9]. But adding the gravitational Chern–Simons term produces a propagating massive graviton [10]. The resulting theory is called topologically massive gravity (TMG). The authors of [11] have done the generalization of topologically massive gravity to higher spins, specifically spin-3 (see also [12]). In this paper we would like to obtain conserved charges of black hole solutions of spin-3 topologically massive gravity by using a quasi-local formalism. Here, we just consider diffeomorphism covariance and we find energy, angular momentum, and entropy, but we don't investigate the higher-spin charges.

A method to calculate the energy of asymptotically AdS solution was given by Abbott and Deser [13]. Deser and Tekin have extended this approach to the calculation of the energy of asymptotically dS or AdS solutions in higher curvature gravity models and also to TMG [14]. The authors of [15] have obtained the quasi-local conserved charges for black holes in any diffeomorphically invariant theory of gravity. By considering an appropriate variation of the metric, they have established a one-to-one correspondence between the ADT approach and the linear Noether expressions. They have extended this work to a theory of gravity containing a gravitational Chern–Simons term in [16], and have computed the off-shell potential and quasi-local conserved charges of some black holes in TMG. In the first order formalism, quasi-local conserved charges of Lorentz-diffeomorphism covariant theories of gravity are investigated in [17]. But there are theories which are not Lorentz-diffeomorphism covariant so for such theories a method for which one can calculate conserved charges of Lorentz-diffeomorphism covariant theories is useless. In previous paper [19] by introducing the total variation of a quantity due to an infinitesimal Lorentz-diffeomorphism transformation, we have obtained the conserved charges in Lorentz-diffeomorphism non-covariant theories. The formalism proposed in [19] is for on-shell case. In this paper we want to extend this formalism to the spin-3 topologically massive gravity. We find an expression for ADT conserved current which is off-shell conserved for a given Killing vector field. Then we can find the generalized ADT conserved charge by virtue of the Poincare lemma. In Ref. [18] the energy of the higher spin black hole solutions of ordinary higher spin gravity was obtained by canonical formalism. Here we not only obtain the energy but also we find the angular momentum of black hole solutions of spin-3 topologically massive gravity. Furthermore we obtain a general formula for entropy of black hole solutions of spin-3 topologically massive gravity, where by substituting ${e_\mu}^{ab} = {\omega_\mu}^{ab} = 0$, this formula reduces to the entropy formula in the ordinary topologically massive gravity which we obtained previously in the paper [19].