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Non-renormalization of the $V\bar{c}c$-vertices in $\mathcal{N}=1$ supersymmetric theories

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Abstract

Using the Slavnov–Taylor identities we prove that the three-point ghost vertices with a single line of the quantum gauge superfield are not renormalized in all loops in $\mathcal{N}=1$ supersymmetric gauge theories. This statement is verified by the explicit one-loop calculation made by the help of the BRST invariant version of the higher covariant derivative regularization. Using the restrictions to the renormalization constants which are imposed by the non-renormalization of the considered vertices we express the exact NSVZ β-function in terms of the anomalous dimensions of the Faddeev–Popov ghosts and of the quantum gauge superfield. In the expression for the NSVZ β-function obtained in this way the contributions of the Faddeev–Popov ghosts and of the matter superfields have the same structure.

1. Introduction

Existence of ultraviolet divergences is a long standing problem of quantum field theory. Supersymmetry allows to improve the ultraviolet behavior due to the so-called non-renormalization theorems. For example, $\mathcal{N}=4$ supersymmetric Yang–Mills (SYM) theory is finite in all orders [1–4], and $\mathcal{N}=2$ supersymmetric theories are divergent only in the one-loop approximation [1,4,5]. Using the $\mathcal{N}=2$ non-renormalization theorem it is possible to construct finite theories with $\mathcal{N}=2$ supersymmetry [6]. It is well known that the superpotential of $\mathcal{N}=1$ supersymmetric

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theories does not receive divergent quantum corrections [7], and the β-function of these theories is related to the anomalous dimension by a special equation [8–13], which is called the exact NSVZ β-function (or the NSVZ relation). For the $\mathcal{N}=1$ SYM theory without matter superfields the NSVZ equation gives the exact expression for the β-function, which appears to be a geometric progression.

The non-renormalization theorems appear due to large symmetries of a theory. Therefore, deriving them it is essential to assume that these symmetries remain unbroken at the quantum level. This means that one has to use an invariant regularization.[1] In supersymmetric theories it is not a trivial problem [18], because the dimensional regularization [19–22] breaks the supersymmetry [23], while its modification called the dimensional reduction [24] is not mathematically consistent [25]. Removing the inconsistencies leads to the loss of manifest supersymmetry [26] and to breaking supersymmetry by quantum corrections in higher loops [27–29]. Actually, the only invariant regularization which can keep supersymmetry and the gauge invariance unbroken is the higher covariant derivative regularization [30,31]. In the supersymmetric case it can be formulated in the manifestly supersymmetric way in terms of $\mathcal{N}=1$ superfields [32,33]. It was also generalized to the case of $\mathcal{N}=2$ supersymmetry [34,35], but in order to have manifest $\mathcal{N}=2$ supersymmetry at all steps of quantum corrections calculating one should formulate the higher derivative regularization in $\mathcal{N}=2$ harmonic superspace [36,37]. This was done in [38] and allows to give a simple proof of the $\mathcal{N}=2$ non-renormalization theorem.

In this paper we investigate renormalization of theories with $\mathcal{N}=1$ supersymmetry, so that we will use the $\mathcal{N}=1$ supersymmetric BRST invariant version of the higher covariant derivative regularization. This regularization allows to calculate quantum corrections in a manifestly gauge and $\mathcal{N}=1$ supersymmetric way. An example of such a calculation can be found in [39], where the one-loop divergences have been obtained using this regularization. The result reveals an interesting feature of the quantum corrections: the three-point vertices with two ghost legs and one leg of the quantum gauge superfields are finite in the one-loop approximation. In this paper we prove that this fact is not accidental and follows from the Slavnov–Taylor identities [40,41] for the general renormalizable $\mathcal{N}=1$ supersymmetric gauge theory with matter. In principle, this statement can be considered as a new non-renormalization theorem in $\mathcal{N}=1$ supersymmetric theories. Moreover, it seems to be useful for deriving the exact NSVZ β-function by the direct summation of Feynman diagrams in the non-Abelian case.

In the Abelian case the NSVZ relation was obtained by the direct summation of Feynman diagrams in all orders for the renormalization group (RG) functions defined in terms of the bare coupling constant in [42,43]. A similar expression for the Adler D-function [44] in $\mathcal{N}=1$ SQCD was also derived in [45,46]. Both these derivations are based on the observation that the integrals giving the β-function (defined in terms of the bare coupling constant) in supersymmetric theories are integrals of (double) total derivatives in the momentum space [47,48]. This structure of loop integrals was confirmed by a large number of explicit loop calculations (see, e.g. [49–53,35, 38]). It allows calculating one of the loop integrals analytically and relating renormalization of the coupling constant in a certain order to the renormalization of the matter superfields in the previous order. Qualitatively this picture is illustrated by Fig. 1 [48,53,54]. From the left we present two-loop diagrams contributing to the β-function. They contain two external lines of the background gauge superfield attached to the same two-loop vacuum graph, which is shown in

[1] Non-invariant regularizations supplemented by a special subtraction scheme which restore the Slavnov–Taylor identities can be also used [14–17], but they are much more inconvenient.