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Lorentz-diffeomorphism quasi-local conserved charges and Virasoro algebra in Chern–Simons-like theories of gravity

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Abstract

The Chern–Simons-like theories of gravity (CSLTG) are formulated at first order formalism. In this formalism, the derivation of the entropy of a black hole on bifurcation surface, as a quasi-local conserved charge is problematic. In this paper we overcome these problems by considering the concept of total variation and the Lorentz–Lie derivative. We firstly find an expression for the ADT conserved current in the context of the CSLTG which is based on the concept of the Killing vector fields. Then, we generalize it to be conserved for all diffeomorphism generators. Thus, we can extract an off-shell conserved charge for any vector field which generates a diffeomorphism. The formalism presented here is based on the concept of quasi-local conserved charges which are off-shell. The charges can be calculated on any codimension two space-like surface surrounding a black hole and the results are independent of the chosen surface. By using the off-shell quasi-local conserved charge, we investigate the Virasoro algebra and find a formula to calculate the central extension term. We apply the formalism to the BTZ black hole solution in the context of the Einstein gravity and the Generalized massive gravity, then we find the eigenvalues of their Virasoro generators as well as the corresponding central charges. Eventually, we calculate the entropy of the BTZ black hole by the Cardy formula and we show that the result exactly matches the one obtained by the concept of the off-shell conserved charges.

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1. Introduction

As is well known, the concept of conserved charges of gravity theories is related to the concept of the Noether charges corresponding to the Killing vectors which are admitted by solutions of a theory. There are several approaches to obtain the mass and angular momentum of black holes solutions of different gravity theories [1–15]. According to the Arnowitt, Deser, and Misner method (ADM method) [2] one can obtain the conserved charges of an asymptotically flat spacetime solution of a general theory of relativity, but this is not a covariant method. The ADM method has been extended to include asymptotically AdS spacetime solution of Einstein gravity [1]. Deser and Tekin have extended this approach. By this extension one can calculate the energy of asymptotically dS or AdS solutions in higher curvature gravity models and also in a topologically massive gravity model (TMG) [3]. This method is a covariant formalism; it is known as the Abbott–Deser–Tekin (ADT) formalism. Another method is the Brown–York approach [4] which is based on a quasi-local concept, but this approach also is not covariant. The authors of [7] have computed the ADT charges for a solution of TMG linearized about an arbitrary background and have applied the result to evaluate the mass and angular momentum of the non-asymptotically flat, non-asymptotically AdS black hole solution (ACL black hole) of TMG.

A general definition of conserved charges in general relativity and other theories of gravity has been proposed in [16]. In the metric formalism of gravity for the covariant theories defined by a Lagrangian n-form L, Wald has shown that the entropy of black holes is the Noether charge associated with the horizon-generating Killing vector field evaluated at the bifurcation surface. Tachikawa extended the Wald approach to include non-covariant theories [17]. Hence, regarding this extension one can calculate the black hole entropy as a Noether charge in the context of non-covariant theories as well. But it is clear now that the derivation of the classical Wald formula for entropy is problematic in the first order formalism using the spin connection. In the first order formalism, the expression of conserved charges are proportional to the Killing vector field ξ. It is clear that ξ must be zero on the bifurcation surface when we calculate the entropy of black hole, because ξ is the horizon-generating Killing vector field which is zero on the bifurcation surface. It seems disappointing at the first glance because it appears that the entropy will be zero, but it is not true. On the other hand there is a class of gravitational theories in $(2+1)$-dimension (e.g. Topological massive gravity (TMG) [18], New massive gravity (NMG) [19], Generalized Massive Gravity (GMG) [20], Minimal massive gravity (MMG) [21], Zewidreibein gravity (ZDG) [22], Generalized minimal massive gravity (GMMG) [23], etc.) which are well-known as the Chern–Simons-like theories of gravity (CSLTG) [24], and can be written in the first order formalism. The authors of [25] have shown that in approaching the bifurcation surface, the spin-connection diverges in a way that the spin-connection interior product in ξ remains finite ensuring that there is no problem. Recently we have extended this formalism in the on-shell case, to the Lorentz-diffeomorphism non-covariant theories [27]. Here we would like to extend this formalism to the off-shell case in the framework of CSLTG. We derive the conserved charge formula by a new method and explicitly compute conserved charges in some models.

The authors of [5] have obtained the quasi-local conserved charges for black holes in any diffeomorphically invariant theory of gravity. By considering an appropriate variation of the metric, they have established a one-to-one correspondence between the ADT approach and the linear Noether expressions.[1] They have extended this work to a theory of gravity contain-

[1] In quasi-local approach [5,6], the Killing vector field ξ is defined not only in the asymptotical part of space, but also in any other points of the space-time. For example see equation (33) below, where Σ is an arbitrary space-like