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Towards weakly constrained double field theory

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Abstract

We show that it is possible to construct a well-defined effective field theory incorporating string winding modes without using strong constraint in double field theory. We show that X-ray (Radon) transform on a torus is well-suited for describing weakly constrained double fields, and any weakly constrained fields are represented as a sum of strongly constrained fields. Using inverse X-ray transform we define a novel binary operation which is compatible with the level matching constraint. Based on this formalism, we construct a consistent gauge transform and gauge invariant action without using strong constraint. We then discuss the relation of our result to the closed string field theory. Our construction suggests that there exists an effective field theory description for massless sector of closed string field theory on a torus in an associative truncation.

1. Introduction

Recent progress in understanding T-duality through double field theory (DFT) [1–6] has lead to interest in constructing an effective theory describing string winding states. When a string is moving on a torus bundle where the radius of torus is near self-dual radius $R \simeq \sqrt{\alpha'}$, the momentum and winding states are treated symmetrically due to the T-duality. If we focus on the torus fibre, string states are specified by momentum p and winding number w, hence target spacetime fields also depend on both p and w, or x and $\tilde{x}$ which are the periodic coordinates for torus and its dual torus respectively. Such fields are called *double fields* and defined on doubled tori.

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However, double fields are not arbitrary functions with respect to the x and $\tilde{x}$, but they are constrained by level matching constraint

$$\big(L_0 - \bar{L}_0\big)\Phi(x,\tilde{x}) = 0. \tag{1.1}$$

For simplicity, if we consider only massless subsector $N = \bar{N} = 1$, then the level matching constraint reduces to

$$\partial_I \partial^I \Phi(x,\tilde{x}) = 0, \tag{1.2}$$

and the massless fields are defined on a $(2d-1)$-dimensional cone in a doubled momentum space. The effective field theory for massless double fields is called double field theory. It is expected that DFT provides an effective field theory for closed strings on a torus background beyond the conventional supergravities.

However, the full DFT has not been constructed yet. The main obstruction in constructing DFT is that the ordinary product of double fields $f(x,\tilde{x})$ and $g(x,\tilde{x})$ does not satisfy level matching constraint again

$$\partial_I \partial^I (f \cdot g) \neq 0. \tag{1.3}$$

In order to make it possible to satisfy level matching constraint, we require so-called strong constraint that all the fields and the gauge parameters as well as all of their products should be annihilated

$$\partial_I f \cdot \partial^I g = 0, \tag{1.4}$$

and the strongly constrained fields are defined on a maximal null plane which is specified by section condition. Any field satisfying the strong constraint is called a strongly constrained field. By imposing the strong constraint, a consistent gauge transform and gauge invariant action has been constructed in a $\mathbf{O}(d,d)$ covariant form [6]. We will refer to the DFT with fields obeying the strong constraint as strongly constrained DFT to distinguish it from DFT with weakly constrained fields which we will call simply DFT if the strong constraint is not imposed.

Another important issue for weakly constrained DFT is that string massive states cannot be decoupled near self-dual radius. An effective field theory includes the appropriate degrees of freedom to describe physical phenomena occurring at a given energy scale, while ignoring degrees of freedom at shorter distances. However, we cannot keep only massless states in closed string field theory near self-dual radius. In order to get a theory for massless degrees of freedom, we should integrate out all the massive fields by hand. Obviously, DFT is not a usual low energy effective field theory. Such computation is not practically possible, and it is not clear whether effective field theory description is valid.

Recent works have addressed the relaxation of the strong constraint in generalized Scherk–Schwarz reduction [7–16]. It turns out that the generalize twist matrix or generalized frame fields are not necessary to satisfy level matching constraint. However, it has been shown that if we relax the strong constraint, then the weak constraint is also violated [8]. It is not clear whether such backgrounds are well-defined as string backgrounds. It is known for example that such backgrounds violate modular invariance [17].

In the present paper, we show that a full relaxation of strong constraint is possible, and we explicitly construct a well-defined gauge transform and associated gauge invariant action without using the strong constraint. The main ingredient for this construction is the X-ray (or Radon) transform on a torus [18–20]. Usually it is applied to X-ray images in tomography. In the context of DFT, the X-ray transform is used to represent a weakly constrained field on a doubled torus