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$\mathcal{N} = 2^\star$ from topological amplitudes in string theory

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Abstract

In this paper, we explicitly construct string theory backgrounds that realise the so-called $\mathcal{N} = 2^\star$ gauge theory. We prove the consistency of our models by calculating their partition function and obtaining the correct gauge theory spectrum. We further provide arguments in favour of the universality of our construction which covers a wide class of models all of which engineer the same gauge theory. We reproduce the corresponding Nekrasov partition function once the Ω-deformation is included and the appropriate field theory limit taken. This is achieved by calculating the topological amplitudes F_g in the string models. In addition to heterotic and type II constructions, we also realise the mass deformation in type I theory, thus leading to a natural way of uplifting the result to the instanton sector.

1. Introduction

During the last decades, the interplay between string theory and supersymmetric gauge theories has been the driving force for many discoveries in both fields. One of the most striking examples is the connection between topological string theory [1] and the $\mathcal{N} = 2$ gauge theory with the Ω-deformation [2,3]. Indeed, it has been realised that the partition function of the topological string reduces, in the field theory limit, to the free energy of the Ω-deformed $\mathcal{N} = 2$ gauge theory, often referred to as the Nekrasov partition function. This correspondence is valid in the

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so-called topological limit of the Ω-background, in which one of its parameters is set to zero, while the other is identified with the topological string coupling g_s. Therefore, what plays the role of a regularisation parameter in gauge theory acquires physical significance once uplifted to string theory. The extension of this connection to the general setup including two deformation parameters is a programme called *refinement* and has led to many fruitful discoveries [4–7]. In particular, a worldsheet realisation of the Ω-background has been carried out in [8,9], even though the explicit definition of the twisted, topological theory is still lacking.

From the point of view of the physical string in this approach, the Ω-deformation boils down to a background of anti-self-dual graviphotons [10,11] (see also [12]) and self-dual gauge field strengths [8,9]. The gauge theory partition function descends from a class of BPS amplitudes which has been studied both in the $\mathcal{N}=2$ [13–16] as well as in the $\mathcal{N}=4$ case [17,18]. Their BPS nature translates itself into the holomorphic moduli dependence of the corresponding coupling in the string effective action. However, this property is broken at the string level due to boundary effects as expressed, for instance, by the holomorphic anomaly equation [10]. The generalisation of the latter to the refined case has been analysed in [19] (see also [20]) and more recently in [21] from the worldsheet perspective.

In the present work, we study a particular deformation of $\mathcal{N}=4$ Super Yang–Mills theory, commonly referred to as $\mathcal{N}=2^\star$. It corresponds to a mass deformation of the former under which the $\mathcal{N}=2$ adjoint hypermultiplet acquires a mass. Hence, analyticity of the mass parameter renders this theory an interpolation between the pure $\mathcal{N}=4$ and $\mathcal{N}=2$ theories, the latter being recovered as particular limits of zero and infinite mass, respectively. In addition, the $\mathcal{N}=2^\star$ theory is a flagship example in the context of the AGT conjecture [22] which relates it to the two-dimensional Liouville theory on a torus with one puncture playing the role of the massive hypermultiplet. In this correspondence, the Nekrasov partition function is mapped to the Liouville theory conformal block. More general connections can be established by obtaining four-dimensional gauge theories from the $(2,0)$ theory compactified on a genus g Riemann surface with n punctures [23], and considering Liouville theory on the Riemann surface. For instance, a torus with four punctures leads to the $\mathcal{N}=2$, SU(2) gauge theory with four flavours.

Since a worldsheet description of $\mathcal{N}=2^\star$ in string theory is lacking, our goal is to fill this gap by studying string theory with spontaneous breaking of supersymmetry from $\mathcal{N}=4$ to $\mathcal{N}=2$, in such a way that the adjoint hypermultiplet acquires a moduli dependent mass. This is achieved by considering a freely-acting orbifold of the T^6 torus, implementing the uplift of the Scherk–Schwarz mechanism to string theory [24–29]. In this construction, certain states acquire a mass that is inversely proportional to the volume of some cycle of the internal space and the unbroken supersymmetric model is recovered in the limit of zero mass. We first present a heterotic asymmetric orbifold construction which turns out to be the most natural way of realising $\mathcal{N}=2^\star$ in perturbative closed string theory. We show that it indeed provides mass to the adjoint hypermultiplet and correctly reproduces the $\mathcal{N}=2^\star$ spectrum without modifying the gauge group. More generally, we present a correspondence between a wide class of freely-acting orbifolds of $\mathcal{N}=4$ compactifications and the $\mathcal{N}=2^\star$ gauge theory. We elaborate on the universality of this construction and provide evidence supporting the correspondence. In particular, we compute topological amplitudes for these $\mathcal{N}=2^\star$ theories, for symmetric and asymmetric freely acting orbifolds, and confirm that they correctly reduce, in the field theory limit, to the Ω-deformed partition function [30–32] of the $\mathcal{N}=2^\star$ gauge theory. This class of BPS amplitudes, which has proven useful in the study of string dualities, is also analysed in type I theory. The latter turns out to be a natural framework for incorporating gauge theory instanton corrections.