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Nuclear Physics B 909 (2016) 525–541

NUCLEAR PHYSICS B

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T^4 fibrations over Calabi–Yau two-folds and non-Kähler manifolds in string theory

Hai Lin

Yau Mathematical Sciences Center, Tsinghua University, Beijing, 100084, PR China

Received 9 April 2016; accepted 2 June 2016

Available online 8 June 2016

Editor: Leonardo Rastelli

Abstract

We construct a geometric model of eight-dimensional manifolds and realize them in the context of type II string theory. These eight-manifolds are constructed by non-trivial T^4 fibrations over Calabi–Yau two-folds. These give rise to eight-dimensional non-Kähler Hermitian manifolds with $SU(4)$ structure. The eight-manifold is also a circle fibration over a seven-dimensional G_2 manifold with skew torsion. The eight-manifolds of this type appear as internal manifolds with $SU(4)$ structure in type IIB string theory with F_3 and F_7 fluxes. These manifolds have generalized calibrated cycles in the presence of fluxes.

1. Introduction

String theory has elegant and deep mathematical structures. It relates theoretical physics to mathematics and has provided great insights to both areas of research. In particular, a great number of important aspects of geometric questions have occurred and can be addressed in the context of string theory. For instance, manifolds with $SU(n)$ structure, such as the Calabi–Yau n-folds, naturally appear in superstring theory and are important subjects for our understanding.

An interesting model of manifolds with $SU(3)$ structure, is the geometric construction of T^2 fibrations over Calabi–Yau two-folds [1,2]. Such six-dimensional manifolds include not only Calabi–Yau three-folds of the Kähler type, but also non-Kähler Hermitian manifolds with $SU(3)$

E-mail address: hailin@mail.tsinghua.edu.cn.

http://dx.doi.org/10.1016/j.nuclphysb.2016.06.006

structure. They can appear as the internal six-manifolds when the superstring theory is compactified down to four-dimensional spacetime. A natural question that is addressed by this present paper is what happens if we use T^4 fibrations, instead of T^2 fibrations. This corresponds to a geometric model of eight-dimensional manifolds that we construct in this paper.

Internal manifolds with six dimensions have been well-studied, in the context of string compactification. However, eight-dimensional internal manifolds are also very interesting. They can have similar mathematical structures as their six-dimensional counterparts, for example they can be Hermitian and have an $SU(n)$ structure where n is the complex dimension. Furthermore, balanced Hermitian manifolds exist in both six dimensions and eight dimensions. Moreover, eight-manifolds can naturally appear in the compactification of string theory with fluxes to two-dimensional spacetime.

Eight-dimensional manifolds with $SU(4)$ structure include both Kähler Calabi–Yau four-folds and non-Kähler Hermitian manifolds with $SU(4)$ structure. These manifolds are equipped with a Hermitian two-form and a holomorphic four-form. These forms can be constructed by bilinears of internal Killing spinors. These eight-dimensional manifolds have been studied by using the equations of pure spinors in type II string theory [3–5]. The Kähler Calabi–Yau four-folds are the special cases, when both the Hermitian form and holomorphic form are closed. In the presence of fluxes, these forms need not be closed, and this is the case for the non-Kähler $SU(4)$-structure manifolds.

The non-Kähler manifolds can appear naturally in string theory with fluxes. In the compactification of heterotic string theory to four dimensional Minkowski spacetime [6], the internal six-manifolds can become non-Kähler in the presence of fluxes [7,8,1,9]. Various models of constructing heterotic manifolds and their vector-bundles have been put forward [7–13]. They play an important role in searching for realistic string theory vacua with four dimensional Minkowski spacetime.

An interesting type of non-Kähler manifolds, which are very important in differential geometry, are balanced Hermitian manifolds. They are Hermitian manifolds with a Hermitian form and a holomorphic form. For a balanced manifold, unlike Kähler manifolds, its Hermitian form is not closed, however, the $(n-1)$th power of its Hermitian form is closed, where n is the complex dimension of the manifold [14]. Since they impose a weaker condition on the closure of the Hermitian form than the Kähler manifolds, they represent close variants of Kähler manifolds. Some non-Kähler Hermitian balanced manifolds can have trivial canonical bundle, and thus are interesting examples of non-Kähler Calabi–Yau manifolds, see for instance [15]. Moreover, under appropriate blowing-downs or contractions of curves, some classes of balanced manifolds can become Kähler and have projective models in algebraic geometry.

In this paper, we will construct eight-dimensional manifolds of the non-Kähler Hermitian type, by T^4 fibrations over Calabi–Yau two-folds. They have $SU(4)$ structures but are not the standard Kähler Calabi–Yau four-folds. The eight-manifolds can also be viewed as a circle bundle over a seven-dimensional base. We will show that the base is a G_2 manifold with skew torsion. General G_2 manifolds with torsion have been widely studied [16–20]. The geometric model of the eight-manifolds here, fits with type II string theory with F_3 and F_7 fluxes and dilaton, as we will see in the later sections.

The organization of this paper is as follows. In Sec. 2, we construct eight-dimensional Hermitian manifolds by T^4 fibrations over Calabi–Yau two-folds. In Sec. 3, we find that the eight-manifold of this type can be viewed as a circle bundle over a seven-dimensional G_2 manifold with skew torsion. After that in Sec. 4, we find that the eight-manifold of this kind can be used in type IIB string theory on the warped product of a two-dimensional Minkowski spacetime and