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Phase structures of 4D stringy charged black holes in canonical ensemble

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Abstract

We study the thermodynamics and phase structures of the asymptotically flat dilatonic black holes in 4 dimensions, placed in a cavity *a la* York, in string theory for an arbitrary dilaton coupling. We consider these charged black systems in canonical ensemble for which the temperature at the wall of and the charge inside the cavity are fixed. We find that the dilaton coupling plays the key role in the underlying phase structures. The connection of these black holes to higher dimensional brane systems via diagonal (double) and/or direct dimensional reductions indicates that the phase structures of the former may exhaust all possible ones of the latter, which are more difficult to study, under conditions of similar settings. Our study also shows that a diagonal (double) dimensional reduction preserves the underlying phase structure while a direct dimensional reduction has the potential to change it.

1. Introduction

It is known that an asymptotically flat black hole is thermodynamically unstable due to its Hawking radiation. There are in general two ways to resolve the instability issue. One is to add a negative cosmological constant to the system such that the resulting black hole is in AdS space [1]. The other is to place the asymptotically flat spherical black hole in a spherical cavity

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outside its horizon with the temperature of the cavity wall fixed *a la* York [2]. Our interest in this paper is for the latter case. Further when the charge inside the cavity is fixed, we define an canonical ensemble while the fixed potential [3] on the wall of the cavity defines the so-called grand canonical ensemble.

We will study the thermodynamics and phase structures of various 4-dimensional asymptotically flat black holes in string theory given in [4] with an arbitrary dilaton coupling, following the aforementioned cavity approach in canonical ensemble. We ask the following questions: if the temperature at the wall of the cavity and the charge inside it are fixed, what are possible thermally stable phase structures allowed? How do the underlying phase structures depend on the dilaton coupling? What are the possible connections of these phase structures to those of branes in 10 dimensional string theory and 11 dimensional M-theory? Does the dimensional reduction (either direct or diagonal (double) dimensional one) change or preserve the underlying phase structure? We will explore all in this paper.

We first examine the chargeless case. We find that the dilaton coupling has no effect on the phase structure for this case and the underlying phase structure remains precisely the same as that of non-dilatonic case studied before in [5] and that of higher dimensional chargeless branes [6]. In other words, there exists a minimal temperature below which the only thermally stable phase inside the cavity is the so-called hot empty space. Above this minimal temperature, we have in general a small and a large black holes but only the large one is locally stable. Further there exists a transition temperature above the minimal one at which the large locally stable black hole has zero Helmholtz free energy, just as the hot empty space at the same temperature. As such the two phases can coexist and the phase transition between the two is just a first order one since the transition involves an entropy change. For temperature lower than this but still higher than the minimal one, the locally stable black hole has a positive free energy, therefore it is globally unstable and will make a so-called Hawking-Page transition to the hot empty space at the same temperature. Only for the temperature higher than the transition temperature, the locally stable black hole has a negative Helmholtz free energy and becomes a globally stable phase.

We then examine the charged case. We find that when the dilaton coupling is greater than one, the underlying phase structure resembles the one of the chargeless case except that the hot empty space is now replaced by the corresponding extremal black hole with the preset temperature. In particular, this type of phase structure is also essentially the same as that of charged 6-brane case studied in [6]. When the dilaton coupling is equal to unity, this situation resembles that of charged 5-brane case studied also in [6] for which there exists a critical charge but there is no critical second-order phase transition. There are three sub-cases but the phase structure for each of them resembles that of chargeless case, without the presence of the van-der Waals–Maxwell gas–liquid type one, again with the replacement of hot empty space by the corresponding extremal black hole. However, when the dilaton coupling is less than one, we find that there exists a critical charge[1] q_c above which we have a globally stable black hole at every preset temperature. Below this critical charge, we find maximal and minimal temperatures between which there exist three black hole phases. The largest and smallest black holes are locally stable as their free energies are local minima while the intermediate size black hole is unstable as its free energy is a local maximum. Further the free energies of the two locally stable black holes are different and there exists a transition temperature, for any given charge $q < q_c$, at which their free energies are the same. At this temperature, the small and the large black holes can coexist and

[1] Without loss of generality, we assume $q \geq 0$ from now on.