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# Inflationary potentials from the exact renormalisation group

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**Abstract**

We show that an inflationary slow-roll potential can be derived as an IR limit of the non-perturbative exact renormalisation group equation for a scalar field within the mean-field approximation. The result follows without having to specify a Lagrangian in the UV, which we take to be somewhere below the Planck scale to avoid discussing quantum gravity effects. We assume that the theory contains a scalar mode with suppressed coupling to other fields, and that higher derivative couplings are suppressed. In this framework the exact RG equation becomes a one-dimensional Schrödinger equation, which we solve. The effective IR potential is then dominated by the eigen-states of the RG Hamiltonian with the highest eigenvalues. We find that these potentials can generically give rise to slow-roll inflation, which is fully consistent with recent observations. As an example of how the proposed renormalisation group procedure works, we perform an explicit calculation in the $\phi^4$ theory in an appendix.


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## 1. Introduction

Inflation [1–3] is an exponentially fast expansion in the early universe that is claimed to resolve a number of problems of the standard Big Bang cosmology (e.g. the horizon, flatness, smoothness, relic problems). It also provides a mechanism for generation of the density perturbations [4] that have left an imprint on the cosmic microwave background and grown into the observed large-scale structure of the universe [5]. Models of inflation are typically discussed within the slow-roll paradigm, where an inflaton field evolves along a nearly flat potential to its minimum, during which it sources a quasi-exponential growth of the cosmological scale factor. Hence, in the slow-roll regime, the spatial and time derivatives are negligible compared to the approximately constant vacuum energy.

Many potentials driving the inflationary fields have been proposed that are either phenomenological or inspired by various UV theories [6]. In this paper, we propose a different approach to deriving a scalar field theory suitable for the slow-roll inflation: We will argue that inflation can be understood as a very generic, non-perturbative prediction of the exact renormalisation group (ERG), in a way that is largely insensitive to the details of the UV physics which we take to be at a scale somewhere below the Planck scale, $M_{UV} < M_{Pl}$, in order to ignore quantum gravity effects. We add that the observation that renormalisation group dynamics can produce experimentally viable inflation scenarios has been noted in the literature before [7,8]. Moreover, exact RG has also been applied in the context of inflation before, see e.g. [9–11].

To have a valid potential for the description of inflation, we must ensure that the effective theory is valid below some scale $M$, where $M$ is roughly on the order of the inverse Hubble radius at the start of inflation. This is the IR of our theory. Thus, $1/M^4$ will be approximately the volume of the patch of spacetime of the resulting effective theory, which we assume resides within the Hubble radius. We will assume that the UV theory at $M_{UV} >> M$ contains a scalar mode $\phi$ that couples very weakly to other fields. Hence, during the RG flow from $M_{UV}$ to $M$, the corrections to the effective potential of such a scalar from the couplings to other UV fields would remain small. In the RG analysis, $\phi$ can therefore be treated independently.

The second assumption will be the validity of the mean-field approximation (MFA) near the scale $M$.[1] That is, we will treat the constant mode of the theory separately when we do our IR analysis. In the paper, we will give an argument for the plausibility of this assumption, given that a field can be expanded around its constant value. We work in the local potential approximation, where higher derivative couplings are ignored. The argument will then rely on the particular form of the ERG equation. As a result, we will only be interested in the constant mode of the theory, which will be sufficient to describe the effective potential. Without justification, this assumption may at first seem peculiar, but we note that for inflation to begin, the inflaton field has to be sufficiently smooth to overcome the gradient energy preventing the exponential expansion. Indeed, the MFA is a common feature of many other inflationary scenarios, where the field is assumed constant at the scale where inflation begins [15].

Lastly, we will also ignore effects of gravity in the ERG. We start at a scale $M_{UV}$ somewhere below the Planck scale, so that quantum gravity effects can be ignored. For the most part of the RG flow, we are at an energy scale where the vacuum energy is negligible in comparison, and we can work in Minkowski space where gravitational effects are unimportant. Indeed, the de Sitter value of the vacuum energy of the universe is roughly comparable with the scale at

[1] For other recent implementations of the ERG within the MFA, in particular in de Sitter space, see [12–14].