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Electric–magnetic dualities in non-abelian and non-commutative gauge theories

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Abstract

Electric–magnetic dualities are equivalence between strong and weak coupling constants. A standard example is the exchange of electric and magnetic fields in an abelian gauge theory. We show three methods to perform electric–magnetic dualities in the case of the non-commutative $U(1)$ gauge theory. The first method is to use covariant field strengths to be the electric and magnetic fields. We find an invariant form of an equation of motion after performing the electric–magnetic duality. The second method is to use the Seiberg–Witten map to rewrite the non-commutative $U(1)$ gauge theory in terms of abelian field strength. The third method is to use the large Neveu Schwarz–Neveu Schwarz (NS–NS) background limit (non-commutativity parameter only has one degree of freedom) to consider the non-commutative $U(1)$ gauge theory or D3-brane. In this limit, we introduce or dualize a new one-form gauge potential to get a D3-brane in a large Ramond–Ramond (R–R) background via field redefinition. We also use perturbation to study the equivalence between two D3-brane theories. Comparison of these methods in the non-commutative $U(1)$ gauge theory gives different physical implications. The comparison reflects the differences between the non-abelian and non-commutative gauge theories in the electric–magnetic dualities. For a complete study, we also extend our studies to the simplest abelian and non-abelian p-form gauge theories, and a non-commutative theory with the non-abelian structure.

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1. Introduction

The M-theory provides useful dualities to unify different kinds of theories and helps us to understand supergravity solutions [1]. In low-energy limit, the ten dimensional supergravity has the T-duality and S-duality. The T-duality is a duality on a target space. The T-duality of closed string theory [2,3] exchanges the momentum and winding modes, and the T-duality of open string theory exchanges the Dirichlet and Neumann boundary conditions. The T-duality requires an isometry on a compact torus, but a generic background does not always have an isometry in closed string theory. In other words, the T-duality maps single valued fields to non-single valued fields and we lose periodicity of the background. Then we obtain the non-geometric flux after performing the T-duality twice in the case of constant H-flux. This mapping gives rise to a problem on quantum dynamics. The solution is to use a double space to construct a well-defined transition function as a diffeomorphism in closed string theory [4–9]. With a global symmetry description, we sacrifice local symmetry in the double space. Local symmetry in the double space is still possible, but difficulties come from the closure of the generalized Lie derivative. This double construction is also extended to open string theory, and has also been applied to cosmology [10–17]. These formulations rely on geometric constructions from the Courant bracket or generalized geometry [18–20]. This bracket comes from the combination of tangent and cotangent bundles. A theory in a double space with the strong constraints (removing additional coordinates) is equivalent to a theory with the Courant bracket. The S-duality is a non-perturbative duality by exchanging the strong and weak coupling constants. In four dimensional electromagnetism, we have an electric–magnetic duality between electric and magnetic fields. This duality is a special case of the S-duality. A problem with the S-duality is that it is hard to be performed exactly due to some issues involving strong couplings. At low-energy level, one successful example is a low-energy effective theory with a non-commutativity parameter (inversely proportional to antisymmetric backgrounds) being a perturbative parameter [21]. The extension of duality from ten dimensional supergravity to eleven dimensional supergravity is the U-duality combining T-duality and S-duality. The manifest U-duality is studied in [22] using extended coordinates.

String theory is described by a two dimensional sigma model. On bulk, the sigma model describes gravity. When we impose the Dirichlet and Neumann boundary conditions on the sigma model, the boundary term comes from the gauge principle. This boundary term gives a picture of open string ending on a D-brane. The ending point of the open string shows the non-commutativity. Non-commutative geometry is naturally hidden in string theory. The low-energy effective theory [21,23–29] of open string is the Dirac–Born–Infeld (DBI) model. In the DBI model, we have the Seiberg–Witten map that maps the commutative theory to the non-commutative theory. In the non-commutative description, the leading order term in the action is a non-commutative $U(1)$ gauge theory with the Moyal product. The Moyal product captures all the effects of the non-commutativity parameters. We find an alternative way to examine the string theory. Now we have many different kinds of non-commutative geometry generalized from the DBI model. This generalization helps us to find more interesting field theories and constrain our low-energy effective field theories from the non-commutative geometry. The first example is the Nambu–Poisson M5 (NP M5) brane theory. This theory describes a M2–M5 system in the large C field background (only three spatial components) on the non-commutative space at low-energy level [23,24]. Based on dimensional reduction, we find a Dp-brane in the large $(p-1)$-form background [25,26] and a Dp-brane in the large NS–NS two-form background. Especially for $p = 3$, the S-duality relation to all orders is found in [21]. According to the dualities, we find the