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Chiral random matrix model at finite chemical potential: Characteristic determinant and edge universality

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Abstract

We derive an exact formula for the stochastic evolution of the characteristic determinant of a class of deformed Wishart matrices following from a chiral random matrix model of QCD at finite chemical potential. In the WKB approximation, the characteristic determinant describes a sharp droplet of eigenvalues that deforms and expands at large stochastic times. Beyond the WKB limit, the edges of the droplet are fuzzy and described by universal edge functions. At the chiral point, the characteristic determinant in the microscopic limit is universal. Remarkably, the physical chiral condensate at finite chemical potential may be extracted from current and quenched lattice Dirac spectra using the universal edge scaling laws, without having to solve the QCD sign problem.

1. Introduction

QCD breaks spontaneously chiral symmetry with a wealth of evidence in hadronic processes at low energies [1]. First principle lattice simulations strongly support that [2]. The spontaneous

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breaking is characterized by a large accumulation of eigenvalues of the Dirac operator near zero-virtuality [3]. The zero virtuality regime is ergodic, and its neighborhood is diffusive [4].

The ergodic regime of the QCD Dirac spectrum is amenable to a chiral random matrix model [5]. In short, the model simplifies the Dirac spectrum to its zero-mode-zone (ZMZ). The Dirac matrix is composed of hopping between N-zero modes and N-anti-zero modes because of chirality, which are Gaussian sampled by the maximum entropy principle. The model was initially suggested as a null dynamical limit of the random instanton model [6]. It also follows from the ϵ-expansion in QCD [7].

QCD at finite chemical potential μ is subtle on the lattice due to the sign problem [8]. A number of effective models have been proposed to describe the effects of matter in QCD with light quarks [1]. Chiral random matrix models offer a simple construct that retains some essentials of chiral symmetry both in vacuum and matter. For instance, in the chiral 1-matrix model finite μ is captured by a constant deformation of Gaussian matrix ensembles [9,10]. In the chiral 2-matrix model the deformation with μ is also random [11,12]. Chiral matrix models in matter were discussed by many [13,14]. Recently both a universal shock analysis [15] and a hydrodynamical description of the Dirac spectra were suggested [16] both at zero and finite chemical potential.

The matrix models were shown to exhibit the same microscopic universality for small eigenvalues in the ergodic regime with vanishingly small μ^2 in the large volume limit [14]. The chief observation is that in the weakly non-hermitean limit, the matrix models can be deformed in a way that preserves the global aspects of the coset manifold under the general strictures of spontaneously broken chiral symmetry and power counting in the so-called epsilon-regime [17].

At finite μ the distribution of Dirac eigenvalues in the complex plane maps onto a 2-dimensional Coulomb gas whose effective action is mostly controlled by Coulomb's law, the conformal and gravitational anomalies in 2-dimensions [16]. These constraints on the Dirac spectrum are beyond the range of chiral symmetry. The eigenvalues form Coulomb droplets that stretch and break at finite μ. The accumulation of the complex eigenvalues at the edge of the droplet may signal a new form of universality unknown to chiral symmetry. The purpose of this paper is to explore this possibility using the concept of characteristic determinants for a unitary random matrix model at finite μ.

With this in mind, we start by developing a stochastic evolution for a Wishart characteristic determinant associated to the standard chiral random matrix model for QCD Dirac spectra at finite chemical potential μ, much along the lines suggested in [15] for the Ginibre ensemble. At finite μ the eigenvalues of the Dirac operator spread in the complex plane. Their accumulation in droplets break spontaneously holomorphic symmetry [9,10]. The characteristic determinant acts as an order parameter for this breaking being zero within the droplet and finite outside. The evolution involves the eigenvalues as complex masses and their conjugates and is diffusion-like asymptotically. The universal behavior of the characteristic determinant at the edge of the Ginibre droplet observed in [15] will be exploited here to derive a universal edge behavior for the Dirac spectra at finite chemical potential.

Finally, we note that the study of deformed and non-hermitean Wishart matrices is interesting on its own as it is of interest to many other areas such as telecommunications and finances, where issues of signal to noise in the presence of attenuation or losses are relevant in designing more efficient routers or financial instruments [18].

The main and new results of the paper are the following: 1) The derivation of a closed evolution equation for the characteristic determinant for a non-hermitean deformation of Wishart matrices in relation to a 1-matrix model for the phase quenched QCD with $N_f = 2$ flavors at finite μ; 2) An explicit derivation of the envelope of the complex eigenvalues for the deformed