Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 907 (2016) 572–596

NUCLEAR PHYSICS B

www.elsevier.com/locate/nuclphysb

Analytical study of Yang–Mills theory in the infrared from first principles

Fabio Siringo

Dipartimento di Fisica e Astronomia dell'Università di Catania, INFN Sezione di Catania, Via S. Sofia 64, I-95123 Catania, Italy

Received 15 February 2016; received in revised form 29 March 2016; accepted 15 April 2016

Available online 20 April 2016

Editor: Tommy Ohlsson

Abstract

Pure Yang–Mills SU(N) theory is studied in the Landau gauge and four dimensional space. While leaving the original Lagrangian unmodified, a double perturbative expansion is devised, based on a *massive* free-particle propagator. In dimensional regularization, all diverging mass terms cancel exactly in the double expansion, without the need to include mass counterterms that would spoil the symmetry of the Lagrangian. No free parameters are included that were not in the original theory, yielding a fully analytical approach from first principles. The expansion is safe in the infrared and is equivalent to the standard perturbation theory in the UV. At one-loop, explicit analytical expressions are given for the propagators and the running coupling and are found in excellent agreement with the data of lattice simulations. A universal scaling property is predicted for the inverse propagators and shown to be satisfied by the lattice data. Higher loops are found to be negligible in the infrared below 300 MeV where the coupling becomes small and the one-loop approximation is under full control.

1. Introduction

In modern textbooks on QCD, the infrared domain is usually called *non-perturbative* just because standard perturbation theory breaks down at the low-energy scale $\Lambda_{QCD} \approx 200$ MeV. While the high energy behavior of the theory is under control and an analytical study of non-

E-mail address: fabio.siringo@ct.infn.it.

http://dx.doi.org/10.1016/j.nuclphysb.2016.04.028
0550-3213/

Abelian gauge theories is usually achieved by a loop expansion in the UV, no analytical first-principle description of the infrared can be found in books where the subject is usually discussed by phenomenological models that rely on numerical lattice simulations.

In the last years, important progresses have been achieved by non-perturbative approaches based on Schwinger–Dyson equations (SDE) [1–9], variational methods [10–20], Gribov copies [21–23] and by simulating larger and larger lattices [24–28] of course. While we still miss a full analytical description, the numerical solution of truncated sets of SDE integral equations together with the *measures* that come from the lattice yield a more clear picture of the infrared behavior of QCD and Yang–Mills theory.

It is now widely believed that in the Landau gauge the gluon propagator is finite and an effective coupling can be defined that is infrared safe and relatively small. As discussed by Cornwall [30] in 1982, the gluon may acquire a dynamical mass in the infrared without breaking the gauge invariance of the theory. The effect cannot be described by the standard perturbation theory at any finite order because of gauge invariance that makes the polarization transverse and prohibits any shift of the pole in the gluon propagator. That is one of the reasons why the standard perturbation theory cannot predict the correct phenomenology in the infrared.

Another reason is the occurrence of a Landau pole in the running of the coupling that makes evident the failure of the perturbative expansion below Λ_{QCD}. However, in the Landau gauge the ghost–gluon vertex function can be shown to be finite [29] and a running coupling can be defined by the product of two-point correlators. Being massive, the gluon propagator is finite and its dressing function vanishes in the infrared yielding a finite running coupling that reaches a maximum and decreases in the low-energy limit [24]. On the other hand, if the Landau pole is an artifact of the perturbative expansion, the relatively small value of the real coupling suggests that we could manage to set up a different perturbative scheme in the infrared. Actually, in order to make physical sense, a perturbative expansion requires that the lowest order term should approximately describe the exact result. While that condition is fulfilled by the standard perturbation theory in the UV, where the propagator is not massive, the dynamical mass of the gluon makes the free propagator unsuitable for describing the low energy limit. Thus we would expect that, by a change of the expansion point, a perturbative approach to QCD in the infrared could be viable.

There is some evidence that inclusion of a mass by hand in the Lagrangian gives a phenomenological model that describes very well the lattice data in the infrared at one loop [31–33]. However that model could be hardly justified by first principles because of the mass that breaks the gauge invariance of the Lagrangian. Even in a fixed gauge, BRST symmetry is broken and a mass counterterm must be included for renormalizing the theory, thus introducing spurious free parameters in the model.

A change of the expansion point can be achieved by first principles without changing the original Lagrangian. Variational calculations have been proposed [16–19] where the zeroth order propagator is a trial unknown function to be determined by some set of stationary conditions. The added propagator is subtracted in the interaction, leaving the total action unchanged. The idea is not new and goes back to the works on the Gaussian effective potential [34–46] where an unknown mass parameter was inserted in the zeroth order propagator and subtracted from the interaction, yielding a pure variational approximation with the mass that acts as a variational parameter. Some recent variational calculations on Yang–Mills theory [18,19] have shown that, provided that we change the expansion point, a fair agreement with the lattice data can be achieved without too much numerical effort. Thus we expect that it is not very important the actual choice of the zeroth order propagator provided that it is massive. A simple free-particle