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Frame-covariant formulation of inflation in scalar-curvature theories

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Abstract

We develop a frame-covariant formulation of inflation in the slow-roll approximation by generalizing the inflationary attractor solution for scalar-curvature theories. Our formulation gives rise to new generalized forms for the potential slow-roll parameters, which enable us to examine the effect of conformal transformations and inflaton reparameterizations in scalar-curvature theories. We find that cosmological observables, such as the power spectrum, the spectral indices and their runnings, can be expressed in a concise manner in terms of the generalized potential slow-roll parameters which depend on the scalar-curvature coupling function, the inflaton wavefunction, and the inflaton potential. We show how the cosmological observables of inflation are frame-invariant in this generalized potential slow-roll formalism, as long as the end-of-inflation condition is appropriately extended to become frame-invariant as well. We then apply our formalism to specific scenarios, such as the induced gravity inflation, Higgs inflation and $F(R)$ models of inflation, and obtain more accurate results, without making additional approximations to the potential. Our results are shown to be consistent to lowest order with those presented in the literature. Finally, we outline how our frame-covariant formalism can be naturally extended beyond the tree-level approximation, within the framework of the Vilkovisky–DeWitt effective action.

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1. Introduction

Inflation, which was originally proposed as a solution to the flatness and horizon problems [1, 2], has been found to be an excellent generic explanation to the origin of anisotropies observed in the cosmic microwave background (CMB) [3–6]. However, the large number of inflationary models underpinned by a variety of theoretical ideas, such as quintessence, modified gravity and string theory, poses a challenge in determining the physical driving mechanism for inflation. Furthermore, due to the complexity of the equations of motion in many inflationary models, extracting analytical predictions for cosmological observables of inflation is rapidly becoming a formidable task. In the simplest of inflationary models, it has been found that if the equations of motion for the classical perturbations of the metric and the inflaton are quantized, the observed tilt of the CMB can be found in terms of solutions to the classical equations of motion. To analytically investigate inflationary models in the general case, the standard procedure is to assume that these solutions satisfy a set of constraints known collectively as the *slow-roll approximation*. In this paradigm, the inflaton field φ is assumed to slowly roll down the inflationary potential $V(\varphi)$, meaning that we may neglect several terms in the equations of motion. Consequently, it is possible to obtain simple analytical expressions for the tilt of the CMB and other inflationary observables.

As observations impose increasingly tighter constraints on inflation [7,8], it becomes difficult to physically motivate minimally coupled inflationary models with acceptable phenomenology both in the context of particle physics and cosmology. A popular alternative is to introduce a coupling function $f(\varphi)$ between the scalar curvature R and the inflaton φ, leading to a more general class of gravity models, termed *scalar-curvature theories*. In these theories, such a coupling function may also be motivated by viewing it as emerging from quantum corrections to the low-energy effective action, after integrating out high-energy degrees of freedom. Thus, it is desirable to extend the procedure for extracting observable quantities from minimally coupled models, in which $f(\varphi) = M_P^2$ where $M_P = 2.435 \times 10^{18}$ GeV is the reduced Planck mass, to general scalar-curvature models, in which $f(\varphi)$ is an arbitrary function of φ. Moreover, scalar-curvature theories with a non-trivial scalar-curvature coupling $f(\varphi)$, which are said to be in the *Jordan frame*, can be recast in the *Einstein frame*, and so be written in terms of minimally coupled models via a combination of conformal transformations and inflaton field reparameterizations. Consequently, studying models related by these transformations can help resolve the so-called *frame problem*, namely whether these models are physically equivalent or not [9–16].

The aim of the present article is to introduce frame covariance in the inflationary dynamics of scalar-curvature theories. This covariance manifests itself as a set of transformation rules that nonetheless keep cosmological observables of inflation invariant. To this end, we develop a new formalism for extracting predictions for observable quantities from scalar-curvature theories by generalizing the corresponding well-known potential slow-roll approximation used in minimally-coupled models. Using this formalism, it is possible to study classes of scalar-curvature theories related to one another by conformal transformations and inflaton field reparameterizations independently. Hence, we will show that these classes of models generate equivalent predictions for inflationary observables. Furthermore, the new formalism may be used as a calculational tool for extracting predictions in a concise and intuitive manner for a wide range of scalar-curvature models without the need for further approximations beyond the ones established in the slow-roll approximation.

The outline of this paper is as follows: after this introductory section, Section 2 introduces the classical action S of the scalar-curvature theories that we will be considering. In particular,