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Three-point disc amplitudes in the RNS formalism

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Abstract

We calculate all tree level string theory vacuum to Dp-brane disc amplitudes involving an arbitrary RR-state and two NS–NS vertex operators. This computation was earlier performed by K. Becker, Guo, and Robbins for the simplest case of a RR-state of type $C^{(p-3)}$. Here we use the aid of a computer to calculate all possible three-point amplitudes involving a RR-vertex operator of type $C^{(p+1+2k)}$.

1. Introduction

D-branes have played many roles in string theory. From the point of view of the string world-sheet they are simply boundary conditions, i.e. strings can end on the D-branes. In practice, this means that if we compute string scattering amplitudes in a background with D-branes (including the type I string, which in this language is interpreted to have space–time-filling D9-branes), then we must include contributions from world-sheets with boundaries, in addition to the usual closed world-sheets.

Alternatively, from the point of view of the low-energy effective theory, D-branes host some degrees of freedom that are localized on the D-brane world-volume. In this paper, we will only be considering separated D-branes, in which case the content of the world-volume theory is

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simply that of maximally supersymmetric super-Yang–Mills with gauge group U(1) for each D-brane. The full effective action is then a sum of a bulk action plus localized actions at each D-brane. These localized actions involve both the world-volume fields and the bulk fields, and can be expanded in derivatives. The details of that expansion are interesting in their own right as an example of an effective theory that admits many different dual perspectives. But even more compellingly, there are examples in which the higher derivative couplings localized on D-branes play an essential role in determining the vacuum structure of string theory, such as in the F-theory duals of M-theory backgrounds on Calabi–Yau four-folds [1,2]. In the IIB description of these constructions, there are D7-branes which wrap four-cycles of the internal space. These D7-branes host four-derivative bulk-field couplings of the schematic form

$$\int_{D7} C^{(4)} \wedge \mathrm{tr}(R \wedge R), \tag{1.1}$$

which lead to the $C^{(4)}$ tadpole equation. This condition is crucial to get consistent solutions. Similarly, there should be more four-derivative couplings which can contribute to charge cancellation in certain other flux backgrounds [3–5]. For these reasons it is important to systematically compute the entire four-derivative effective action localized on a D-brane.

Of course, the world-sheet and effective theory perspectives are related. The terms in the effective action can be computed by the relevant perturbative string scattering amplitudes. For example, the coupling (1.1) can be obtained by computing a three-point disc amplitude with one R–R vertex operator and two graviton vertex operators [6–9] (though there are other methods for deducing these particular couplings [10–14]). As a preliminary step towards computing the full effective action, we need to compute all of the relevant string scattering amplitudes, as we do herein.

We calculate type II superstring scattering amplitudes on world-sheets with the topology of a disk, with closed or open string insertions. We are following references [15], [16], and [3], where the formalism was developed and some simple amplitudes were computed. Similarly as done in these references, the final goal is to extract information about the corresponding Dp-brane effective actions. Some new aspects of these actions are discussed in a forthcoming paper [17].

The calculation of the two-point function involving one R–R state and one NS–NS state appeared in earlier papers [18–22] or in the notation and conventions used herein in [15]. The three-point amplitude involving one R–R field of type $C^{(p-3)}$ was calculated in [16,23], and some pieces for $C^{(p+5)}$ appeared in [23]. Our goal here is to compute the most general tree level string theory vacuum to Dp-brane amplitude with insertion of an arbitrary R–R state and various NS–NS vertex operators. We then restrict to the case of one R–R field and two NS–NS fields. This amplitude is expressed in terms of the R–R potential $C^{(p+1+2k)}$ and two NS–NS fields. The collection of these amplitudes is shown in Fig. 7 of [16] (reproduced here in Fig. 1).

Because the amplitudes are invariant under a certain $\mathbb{Z}_2$ symmetry (combining reflection in the space–time directions that are normal to the brane with worldsheet parity), they are non-vanishing only if

1. k is even and both NS–NS fields are antisymmetric or both are symmetric.
2. k is odd and one of the NS–NS fields is symmetric and the other one is antisymmetric.

In general, the coefficients of the amplitudes cannot be evaluated analytically, so we write them in a complex integral form. Due to conservation of momentum and integration by parts, the