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The odd-order Pais–Uhlenbeck oscillator

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Abstract

We consider a Hamiltonian formulation of the $(2n+1)$-order generalization of the Pais–Uhlenbeck oscillator with distinct frequencies of oscillation. This system is invariant under time translations. However, the corresponding Noether integral of motion is unbounded from below and can be presented as a direct sum of $2n$ one-dimensional harmonic oscillators with an alternating sign. If this integral of motion plays a role of a Hamiltonian, a quantum theory of the Pais–Uhlenbeck oscillator faces a ghost problem. We construct an alternative canonical formulation for the system under study to avoid this nasty feature.

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1. Introduction

The Pais–Uhlenbeck (PU) oscillator [1] is one of the simplest higher-derivative mechanical models. In general, one constructs a Hamiltonian formulation of this system with the aid of Ostrogradsky's method [2]. But the Hamiltonian obtained in such a way is unbounded from below. This leads to a ghost problem on quantization [1,3,4]. The research of higher-derivative theories of gravity stimulates considerable interest in solving this long-standing problem. To obtain a more physically viable quantum theory of the PU oscillator, the efforts have been focused mostly on the construction of alternative Hamiltonian formulations and quantization procedures [5–19]. In particular, in [5] it has been shown that a canonical formulation of the fourth-order PU oscillator is not unique. Moreover, in contrast to Ostrogradsky's approach, alternative canonical formalism may correspond to a positive-definite Hamiltonian.

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According to the analysis in [1] (see also [4]), the Ostrogradsky's Hamiltonian of the $2n$-order PU oscillator with distinct frequencies of oscillation can be presented as a direct sum of n decoupled harmonic oscillators with an alternating sign. This provides a set of n functionally independent positive-definite integrals of motion. The method introduced in [5] is based on observation that a linear combination of these integrals with arbitrary nonzero coefficients can play a role of a Hamiltonian for the case of the fourth-order PU oscillator. So, we have a positive-definite Hamiltonian when all coefficients are positive. The corresponding Poisson bracket can be obtained as a solution of a nondegenerate system of linear equations. The same approach allows to construct an alternative canonical formulation for the PU oscillator of an arbitrary even order with distinct frequencies of oscillation [19].

In recent time the PU oscillator has also attracted much attention within the context of dynamical realizations of nonrelativistic conformal groups [17,20–30]. In particular, it has been shown that the $2n$-order PU oscillator for a particular choice of its frequencies of oscillation enjoys $l = n - \frac{1}{2}$-conformal Newton–Hooke symmetry [31–34]. On the other hand, the analogous dynamical realization of the l-conformal Newton–Hooke algebra for integer $l = n$ represents a higher-derivative model which is naturally called as $(2n + 1)$-order PU oscillator [22,25,26]. Some aspects of the third-order PU oscillator have been studied in papers [35–38] (see also [39–42]). But the odd-order PU oscillator for values of order higher than three remains completely unexplored. At the same time, a construction of a Hamiltonian formulation of this model is an important issue for further possible quantum mechanical applications. The purpose of this work is to generalize the analysis obtained in papers [5,19] to the case of the PU oscillator of the arbitrary odd order with distinct frequencies of oscillation.

The paper is organized as follows. In the next section we give the notion of the odd-order PU oscillator. In Sect. 3, we construct an alternative Hamiltonian formulation for the third-order PU oscillator. A Hamiltonian formulation for the PU oscillator of the arbitrary odd order is considered in Sect. 4. Sect. 5 is devoted to possible generalizations of the odd-order PU oscillator which are compatible with alternative canonical formalism. In the concluding Sect. 6, we summarize our results and discuss further possible developments. Some technical details are gathered in Appendix A.

2. The model

The recent results on dynamical realizations of so-called l-conformal Galilei algebra [31–33] have shown that such a model as a free $(2l + 1)$-order derivative particle exhibits this symmetry [43] (see also [44–49]).[1] The action functional of this model for half-integer l has a form[2]

$$S = \frac{1}{2}\int dt\, x_i x_i^{(2l+1)}, \tag{1}$$

where $i = 1, 2, .., dim\, V$ is a spatial index; a superscript in braces designates the number of derivatives with respect to time. While for integer l one has

$$S = \frac{1}{2}\int dt\, \epsilon_{ij}\, x_i x_j^{(2l+1)}, \tag{2}$$

where ϵ_{ij} is the Levi-Civitá symbol with $\epsilon_{12} = 1$; $i, j = 1, 2$.

1 About realizations of l-conformal Galilei algebra without higher derivatives see [20,28,50,51].

2 The summation over repeated spatial indices is understood, unless otherwise is explicitly stated.

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