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Vertex operators of ghost number three in Type IIB supergravity

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Abstract

We study the cohomology of the massless BRST complex of the Type IIB pure spinor superstring in flat space. In particular, we find that the cohomology at the ghost number three is nontrivial and transforms in the same representation of the supersymmetry algebra as the solutions of the linearized classical supergravity equations. Modulo some finite dimensional spaces, the ghost number three cohomology is the same as the ghost number two cohomology. We also comment on the difference between the naive and semi-relative cohomology, and the role of b-ghost.

1. Introduction

Vertex operators are one of the central objects in string theory. They represent cohomology classes of the BRST operator. The BRST cohomology depends on the chosen background, and in fact describes the tangent space to the moduli space of backgrounds at the chosen point.

In particular, let us look at the pure spinor superstring theory in expansion around flat space. The structure of massless BRST cohomology in flat space is more or less clear, but it appears

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that it has never been explicitly spelled out in the literature. The present paper is aimed at filling this gap.

For the closed *bosonic* string the cohomology was computed in [1]. We will here do a similar computation for the pure spinor superstring, but with the following difference. It is well known that the physically relevant cohomology problem is the so-called semirelative cohomology [2], which is Q_{BRST} acting on the vertex operators V satisfying the following condition:

$$(b_0 - \overline{b}_0)V = 0 \tag{1}$$

This condition was built-in into the computations of [1]. In the pure spinor superstring, the construction of the b-ghost is very subtle. In our paper we will compute the "naive" cohomology of Q_{BRST}, without taking into account (1). Failure to take into account (1) leads to some strange results:

1. Nonphysical vertex operators, *i.e.* elements of the BRST cohomology which do not correspond to any linearized SUGRA solutions
2. Absence of the dilaton zero mode
3. Nontrivial cohomology at the ghost number three

Problems 1 and 2 are removed if we require the existence of the dilaton superfield Φ (see [3] and the discussion in Section 7.3). To defeat the ghost number three cohomology is more difficult. It is dangerous as a potential obstacle for continuing an infinitesimal solution to a finite solution (*i.e.* obstructed deformations of the flat spacetime). Such obstructions would render the theory physically inconsistent. In bosonic string, all linearized deformations are unobstructed. One explanation is that the *semi-relative* cohomology at the ghost number three is zero, and therefore there is no obstacle. More precisely, the higher order correction to V are controlled by the string field equation [4,5]:

$$QV = (b_0 - \overline{b}_0)(V^2) + \dots \tag{2}$$

Since the ghost number four cohomology is zero, V^2 is in the image of Q. In fact, the pre-image could be chosen to be annihilated by $L_0 - \overline{L}_0$, and this shows that Eq. (2) can be resolved order by order in the deformation parameter.

Unfortunately, we do not have such a proof in the pure spinor formalism. It follows from the consistency of [6] that there is actually no obstacle in extending the infinitesimal deformation to higher orders. Even though the ghost number three cohomology is nonzero, the actual obstruction vanishes for physical states. It would be good to have a transparent proof of this fact using the language of BRST cohomology and vertex operators. This would probably require the use of the composite b-ghost.

1.1. Plan of the paper

In the rest of this introductory section we will review general facts about the BRST cohomology and its relation to the deformations of the worldsheet sigma-model. Then in Section 2 we will review the cohomology of the classical electrodynamics, and explain how to reduce the cohomology of the Type IIB BRST operator in flat space to the cohomology of electrodynamics. The relation will involve the computation of the cohomology of the algebra of translations with coefficients in the space of solutions of SUSY Maxwell equations (Section 3) and the tensor pro-