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Nuclear Physics B 907 (2016) 542–571

NUCLEAR PHYSICS B

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On the construction of integrated vertex in the pure spinor formalism in curved background

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Received 25 June 2015; received in revised form 26 February 2016; accepted 4 April 2016

Available online 7 April 2016

Editor: Herman Verlinde

Abstract

We have previously described a way of describing the relation between unintegrated and integrated vertex operators in $AdS_5 \times S^5$ which uses the interpretation of the BRST cohomology as a Lie algebra cohomology and integrability properties of the AdS background. Here we clarify some details of that description, and develop a similar approach for an arbitrary curved background with nondegenerate RR bispinor. For an arbitrary curved background, the sigma-model is not integrable. However, we argue that a similar construction still works using an infinite-dimensional Lie algebroid.

1. Introduction

The construction of the worldsheet sigma-model for the Type II superstring in the pure spinor formalism is a fundamental problem. It was more or less solved in [1]. However, we feel that some better understanding is possible. First of all, the sigma-model suggested in [1] is technically very special, and it is not clear why this is the most general solution. In particular, the formulation depends crucially on a special choice of fields. Indeed, the theory is not invariant un-

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http://dx.doi.org/10.1016/j.nuclphysb.2016.04.004

der field redefinitions mixing matter fields with ghosts. It would be desirable to have an axiomatic formulation of the sigma-model. Something along these lines:

- A sigma-model with two nilpotent symmetries, Q_L and Q_R, such that the current of Q_L is holomorphic and the current of Q_R is antiholomorphic, and there are symmetries $U(1)_L$ and $U(1)_R$, such that Q_L and Q_R are appropriately charged under them.

However, we feel that this is not enough; the axiomatics sketched above is probably too weak, although it is enough to correctly describe small deformations of the flat space. The correct axiomatics should somehow encode the singularity of the pure spinor cone.

Also, we believe that the worldsheet sigma-model should be formulated as a problem in cohomological perturbation theory. A small neighborhood of each point in space–time can be approximated by flat space:

$$S = \int d\tau^+ d\tau^- \left(\partial_+ X^\mu \partial_- X^\mu + p_+ \partial_- \theta_L + p_- \partial_+ \theta_R + w_+ \partial_- \lambda_L + w_- \partial_+ \lambda_R\right) \tag{1}$$

with BRST symmetries:

$$Q_L = \lambda_L^\alpha \left(\frac{\partial}{\partial \theta_L^\alpha} + \Gamma^m_{\alpha\beta} \theta_L^\beta \frac{\partial}{\partial x^m}\right) + (\ldots) \frac{\partial}{\partial w_+} \tag{2}$$

$$Q_R = \lambda_R^{\hat\alpha} \left(\frac{\partial}{\partial \theta_R^{\hat\alpha}} + \Gamma^m_{\hat\alpha\hat\beta} \theta_R^{\hat\beta} \frac{\partial}{\partial x^m}\right) + (\ldots) \frac{\partial}{\partial w_-} \tag{3}$$

Then we say that a general background is obtained by the deformation of the action accompanied by some deformation[2] of Q_L and Q_R. The infinitesimal deformations at the linearized level are well known to correspond to the linearized SUGRA waves. They were classified in [3]. However, it was shown also in [3] that there is a potential obstacle to extending the deformations beyond the linearized level. The obstacle is a nonzero cohomology group, namely the ghost number three vertex operators. Without doubt, the obstacle actually vanishes (there is a nonzero cohomology group, but the actual class vanishes). This, however, is not well understood. As we explained in [3], one way to prove the vanishing of the obstacle is to consider the action of the b-ghost in cohomology. The formalism that would allow to do such calculation has not yet been fully developed. The definition of the b-ghost requires including the non-minimal fields which makes the lack of axiomatic formulation even more acute. And the b-ghost is nonpolynomial, opening the possibility that at some order the deformed action will also become non-polynomial.[3] Such questions should be addressed together with the problem of axiomatic formulation of the worldsheet theory.

When we study the pure spinor formalism as a cohomological perturbation theory, one important technical aspect is the relation between integrated and unintegrated vertex operators. The deformation of the action is described by *integrated* vertices:

$$S \to S + \int d\tau^+ d\tau^- \, U \tag{4}$$

[2] As we have shown in [2], the very leading effect will be actually the deformation of Q_L and Q_R leaving S undeformed; this corresponds to the linear dilaton.

[3] We have no doubt that this does not happen, it is just that we don't know how to see this using the cohomological perturbation theory.