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Nuclear Physics B 907 (2016) 617–632

NUCLEAR PHYSICS B

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Vacuum decay in CFT and the Riemann–Hilbert problem

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Received 25 March 2016; accepted 30 March 2016

Available online 5 April 2016

Editor: Leonardo Rastelli

Abstract

We study vacuum stability in $1+1$ dimensional Conformal Field Theories with external background fields. We show that the vacuum decay rate is given by a non-local two-form. This two-form is a boundary term that must be added to the effective in/out Lagrangian. The two-form is expressed in terms of a Riemann–Hilbert decomposition for background gauge fields, and its novel "functional" version in the gravitational case.

1. Introduction

In this article we discuss vacuum decay in $1+1$ dimensional Conformal Field Theories with external fixed background fields. As an example, we consider a theory of massless fermions in $1+1$ dimensions coupled to Abelian, non-Abelian or gravitational background fields. The computation of the vacuum decay rate involves evaluating the effective action, which is given by the logarithm of the determinant of the quantum fields in the fixed background. The pioneer example, due to Schwinger [1], is of fermions in a constant background electric field. The example

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http://dx.doi.org/10.1016/j.nuclphysb.2016.03.039
0550-3213/

we study in our paper is interesting, as we can find formulas for vacuum decay in generic field profiles (which satisfy a few technical assumptions that we state below). Some exact results for generic field profiles were also obtained in [2,3], in $1+1$ dimensional QED.

Let us briefly review a case with no particle production. Consider free massless fermions interacting with a fixed non-Abelian gauge field background. The effective action is obtained by the Gaussian integration over the fermion fields, and is given by a one loop determinant. If the field profile satisfies a "good" behavior, that we specify later, the effective action is real and is expressed [4] in terms of the Wess–Zumino–Novikov–Witten (WZNW) action [5–7]. In this case particles are not created, since the vacuum decay rate is nonzero only when the effective action has an imaginary part.

Our goal is to determine the effective action for background fields that do lead to particle production. In this case, we have to discuss the in/out effective action which has an imaginary part, reflecting vacuum decay. The imaginary piece in the effective action is determined by a careful treatment of the Feynman $i\varepsilon$ prescription in a massless theory.

Our main result is that the effective action is modified by the inclusion of extra boundary terms, which are complex, and whose imaginary part gives the vacuum decay rate. The boundary term is a two-form which appears to be novel. To compute the boundary terms we need a certain Riemann–Hilbert decomposition. While the Abelian and non-Abelian decompositions are standard Riemann–Hilbert problems, the gravitational case has not been considered before. The vacuum decay rate for Abelian background fields is given by the same formula of dissipative quantum mechanics obtained by Caldeira and Leggett [8,9]. Our results generalize their formulas for non-Abelian and gravitational backgrounds.

The rest of the paper is organized as follows. In section 2 we compute the effective action and the new boundary term for an Abelian gauge field and discuss the general logic of the computation, which helps in the more complicated cases. In section 3 we find the effective action and the new boundary term for the non-Abelian gauge field. Finally, in section 4 we find the effective action and the new boundary term in the case of the gravitational field. In Appendix A, we discuss an alternative method of computation of the boundary terms. In Appendix B, we review the gauge-gravity duality between the non-Abelian and gravitational cases [10–12]. Finally, in Appendix C, we show the first perturbative correction to the Caldeira–Leggett formula coming from non-Abelian and gravitational backgrounds.

2. Vacuum decay in an Abelian background

To set the stage, let us look at the Abelian case first. The Lagrangian is

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}(i\partial_{\mu} + A_{\mu})\psi = \bar{\psi}_{-}(i\partial_{+} + A_{+})\psi_{-} + \bar{\psi}_{+}(i\partial_{-} + A_{-})\psi_{+}, \tag{2.1}$$

where the metric is $\eta^{\mu\nu} = (1, -1)$, and we introduced light cone coordinates $x^{\pm} = (x^0 \pm x^1)/\sqrt{2}$. From the Lagrangian it is clear that the left movers ψ_+ and right movers ψ_- are sourced by A_- and A_+ fields, respectively. Therefore, the determinant will split into a right-moving piece, a left-moving piece, and a contact term that ensures gauge invariance [4][1]

$$S(A_+, A_-) = \log\det(\gamma^{\mu}(i\partial_{\mu} + A_{\mu})) = W_+(A_+) + W_-(A_-) - 2\int d^2x A_+ A_- . \tag{2.2}$$

[1] In this section we write the effective action up to an unimportant overall factor $-\frac{e^2}{4\pi}$. In other words, we set $e^2 = -4\pi$. The charge e can be restored by the substitution $A_{\mu} \to eA_{\mu}$.