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Off-shell CHY amplitudes

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Abstract

The Cachazo–He–Yuan (CHY) formula for on-shell scattering amplitudes is extended off-shell. The off-shell amplitudes (amputated Green's functions) are Möbius invariant, and have the same momentum poles as the on-shell amplitudes. The working principles which drive the modifications to the scattering equations are mainly Möbius covariance and energy momentum conservation in off-shell kinematics. The same technique is also used to obtain off-shell massive scalars. A simple off-shell extension of the CHY gauge formula which is Möbius invariant is proposed, but its true nature awaits further study.
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1. Introduction

S-matrix theory was very popular in the late 1950s and early 1960s. It sought to deal more directly with physical observables, and to avoid ultraviolet divergences by staying away from local space-time interactions. Unfortunately, it never got too far because dynamics could not be fully introduced without a Lorentz-invariant interaction Lagrangian density. This problem is now nicely circumvented by the Cachazo–He–Yuan (CHY) scattering theory [1–6], where local Lorentz invariance is supplemented by Möbius invariance of the scattering amplitude in an underlying complex plane. Since its inception, there have been many other papers discussing the

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properties of the scattering equations [7–16], calculations of the amplitude [17–21], its relation to string theory [22–24], the soft and collinear limits [25], and generalization to include massive and/or other particles [20,26–29]. The CHY formula, in its original form, is a tree amplitude for massless particles. In order to implement unitarity, generalization to loop amplitudes [30–37] is required. To facilitate such a generalization and to understand better its connection with local quantum field theory, it is necessary to study the off-shell behavior of these scattering amplitudes. This is what we propose to do in this paper. In Sec. 2, we will extend the CHY on-shell scalar amplitude off-shell to get the amputated Green's functions. We will also use the same technique to extend massless amplitudes to massive scalar amplitudes in Sec. 3, on-shell and off-shell. The on-shell version agrees with the result obtained previously by Dolan and Goddard [20]. The same consideration also yields an off-shell extension of the CHY gauge amplitude, which is Möbius invariant, but the implication of such an extension requires more study as we shall discuss in Sec. 4. Some of the illustrative details are contained in the three appendices.

2. Off-shell massless scalar amplitude

Consider a set of scalar fields ϕ^{ia} in which the first index is in the adjoint representation of some Lie algebra and the second index is in another. If they interact tri-linearly through

$$L_{int} = \frac{1}{3!} f_{ijk} g_{abc} \phi^{ia} \phi^{jb} \phi^{kc}, \tag{1}$$

f's and g's being the structure constants in the Lie algebras, then the Green's function for n particle with momenta $k_i, i = 1 \cdots n$ at the tree level will be a function of products of propagators $\frac{1}{s_{i_1 i_2 \cdots i_m}}$ with $2 \le m \le n-2$ and $s_{i_1 i_2 \cdots i_m} \equiv (k_{i_1} + k_{i_2} + \cdots + k_{i_m})^2$. The coefficients will be a product C_i of $n-2$ f's of the first Lie algebra and another product D_a of $n-2$ g's of the other. For some subsets of indices, they satisfy the Jacobi identities

$$C_i + C_j + C_k = 0, \quad D_a + D_b + D_c = 0. \tag{2}$$

Because of this and because of f and g being totally antisymmetric, only $(n-2)!$ of the C's and $(n-2)!$ of the D's are independent. We can choose an independent set, such that C's are of the form $f_{i_1 j_2 j_3} \cdots f_{j_{n-2} j_{n-1} i_n}$ and D's $g_{a_1 b_2 b_3} \cdots g_{b_{n-2} b_{n-1} a_n}$. The n-particle Green's function will be given as

$$\langle(\phi^{i_1 a_1}(k_1)\phi^{i_2 a_2}(k_2)\cdots\phi^{i_n a_n}(k_n))\rangle \approx \langle C|M|D\rangle, \tag{3}$$

irrespective of whether k_i are on-shell or not. Here $\langle C|$ is a vector formed from the independent set just mentioned and so is the vector $|D\rangle$. M is the $(n-2)! \times (n-2)!$ symmetric propagating matrix given by CHY formula when all the $k_i^2 = 0$. Explicit expressions for $n = 4, 5$ were given earlier by Vaman and Yao [38]. In the next two sections, we shall explicitly solve for the modifications to the scattering functions f_i in the CHY formulas such that M takes exactly the same form even when $k_i^2 \neq 0$ for $n = 4, 5$. Generalization to any n will then be given in what follows.

2.1. Four particles

When all the particles are on shell, the amplitudes are given by [1–6]

$$M^{1234\ 1ij4} = -\frac{1}{2\pi i} \oint \frac{d\sigma_3}{f_3} \frac{\sigma_{124}^2}{\sigma_{1234}\sigma_{1ij4}}, \tag{4}$$

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