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Analytic continuations of 3-point functions of the conformal field theory

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Abstract

It is shown that the general 3-point function $\langle \Phi_c \Phi_b \Phi_a \rangle$, with continuous values of charges a, b, c of a statistical model operators, and the 3-point function of the Liouville model, could all be obtained by successive analytical continuations starting from the 3-point function of the minimal model.

1. Introduction

Recent interest in the 3-point functions $\langle \Phi_c \Phi_b \Phi_a \rangle$ with continuous values of charges a, b, c, which do not satisfy the neutrality conditions of the Coulomb gas minimal models, is, principally, due to recently found realisations of these correlation functions in the context of statistical models, on the lattice: Potts model 3 spin correlation function [1], loop models [2].

On the other side, the interest in the Liouville model correlation function was always present, since 1981 [3].

The Liouville 3-point function was defined in [4,5]. The statistical model general 3-point function (of imaginary Liouville or Coulomb gas) was defined in [6]. See also the related work in [7].

In the present paper we rederive these results somewhat differently, by a sequence of analytical continuations, starting with the minimal model 3-point function [8–10].

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The present study does not provide new results but gives some new methods and provides some unification, hopefully.

2. Analytic continuation of $(1, n)$ operators correlation function towards the general (n', n) operators 3-point function

The structure constant of the $(1, n)$ minimal model subalgebra, which is the 3-point function of $(1, n)$ operators, is of the form [9]:

$$\langle V^{+}_{1,p}(\infty)V_{1,n}(1)V_{1,m}(0)\rangle$$
$$=\prod_{j=1}^{k}\frac{\Gamma(j,\rho)}{\Gamma(1-j\rho)}\times\prod_{j=0}^{k-1}\frac{\Gamma(1+\alpha+j\rho)\Gamma(1+\beta+j\rho)\Gamma(1+\gamma+j\rho)}{\Gamma(-\alpha-j\rho)\Gamma(-\beta-j\rho)\Gamma(-\gamma-j\rho)} \tag{2.1}$$

where $V_{1,m}$, $V_{1,n}$, $V^{+}_{1,p}$, are the Coulomb gas vertex operators,

$$V(z,\bar{z})_{1,m}=V_{\alpha_{1,m}}(z,\bar{z})=e^{i\alpha_{1,m}\varphi(z,\bar{z})},$$
$$V_{1,n}(z,\bar{z})=V_{\alpha_{1,n}}(z,\bar{z})=e^{i\alpha_{1,n}\varphi(z,\bar{z})}$$
$$\alpha_{1,m}=\frac{1-m}{2}\alpha_{+},\quad \alpha_{1,n}=\frac{1-n}{2}\alpha_{+} \tag{2.2}$$

$V^{+}_{1,p}$ is the Coulomb gas conjugate operator:

$$V^{+}_{1,p}(z,\bar{z})=V_{\alpha^{+}_{1,p}}(z,\bar{z})=e^{i\alpha^{+}_{1,p}\varphi(z,\bar{z})}$$
$$\alpha^{+}_{1,p}=2\alpha_0-\alpha_{1,p}=2\alpha_0-\frac{(1-p)}{2}\alpha_{+}=\alpha_{-}+\frac{1+p}{2}\alpha_{+} \tag{2.3}$$

$\varphi(z,\bar{z})$ is the Coulomb gas field.

Parameters $\alpha, \beta, \gamma, \rho$ in (2.1) are defined as:

$$\alpha=2\alpha_{+}\alpha_{1,m}=(1-m)\rho,\quad \beta=2\alpha_{+}\alpha_{1,n}=(1-n)\rho,$$
$$\gamma=2\alpha_{+}\alpha^{+}_{1,p}=2\alpha_{+}(2\alpha_0-\alpha_{1,p})=2\alpha_{+}(\alpha_{-}+\frac{1+p}{2}\alpha_{+})=-2+(1+p)\rho,\quad \rho=\alpha_{+}^{2} \tag{2.4}$$

α_{+}, α_{-} are the charges of the screening operators

$$V_{+}(z,\bar{z})=e^{i\alpha_{+}\varphi(z,\bar{z})},\quad V_{-}(z,\bar{z})e^{i\alpha_{-}\varphi(z,\bar{z})} \tag{2.5}$$

α_0 is the Coulomb gas background charge, $2\alpha_0=\alpha_{+}+\alpha_{-}$, and $\alpha_{+}\cdot\alpha_{-}=-1$.

The parameter k in (2.1) is the number of screening operators V_{+}, required by the function on the l.h.s. of (2.1), to satisfy the neutrality condition:

$$\alpha^{+}_{1,p}+\alpha_{1,n}+\alpha_{1,m}+k\alpha_{+}=2\alpha_0 \tag{2.6}$$

On finds that

$$k=\frac{m+n-p-1}{2} \tag{2.7}$$