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Hagedorn transition and topological entanglement entropy

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Abstract

Induced by the Hagedorn instability, weakly-coupled $U(N)$ gauge theories on a compact manifold exhibit a confinement/deconfinement phase transition in the large-N limit. Recently we discover that the thermal entropy of a free theory on $\mathbb{S}^3$ gets reduced by a universal constant term, $-N^2/4$, compared to that from completely deconfined colored states. This entropy deficit is due to the persistence of Gauss's law, and actually independent of the shape of the manifold. In this paper we show that this universal term can be identified as the topological entangle entropy both in the corresponding $4+1D$ bulk theory and the dimensionally reduced theory. First, entanglement entropy in the bulk theory contains the so-called "particle" contribution on the entangling surface, which naturally gives rise to an area-law term. The topological term results from the Gauss's constraint of these surface states. Secondly, the high-temperature limit also defines a dimensionally reduced theory. We calculate the geometric entropy in the reduced theory explicitly, and find that it is given by the same constant term after subtracting the leading term of $\mathcal{O}(\beta^{-1})$. The two procedures are then applied to the confining phase, by extending the temperature to the complex plane. Generalizing the recently proposed $2D$ modular description to an arbitrary matter content, we show the leading local term is missing and no topological term could be definitely isolated. For the special case of $\mathcal{N}=4$ super Yang–Mills theory, the results obtained here are compared with that at strong coupling from the holographic derivation.

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1. Motivation

In recent years intensive studies on entanglement entropy (EE) show that gauge theories possess nontrivial entanglement pattern [1]. Practically, different patterns of entanglement could then be employed to classify different phases of gauge theory. In the case of discrete gauge groups, topological order (TO) characterizes different phases of the gauge theories [2].[1] More explicitly, the deconfined phase of discrete gauge theory exhibits nontrivial TO. TO is captured by a set of topological data, and one of them is topological entanglement entropy (TEE), the topological term extracted from the large-size expansion of EE [5,6]. A non-vanishing TEE is intimately related to the global gauge-invariant constraint in the deconfined phase, according to the string-net condensation mechanism [7]. Generalization of TO to gauge theory with a continuous gauge group seems extremely urgent, but turns out not so straightforward. In strong-coupled gauge theories with a gravity dual [8–10], the entanglement entropy can be conveniently calculated with the Ryu–Takayanaki formula [11,12], and attains manifestly the area-law term. The holographic EE behaves quite differently at different subregion scales in a confining background [13,14]. When properly extended to finite temperature, it also changes discontinuously across the deconfining temperature [15]. However, the holographic derivation gives a vanishing result for the topological term [16]. It is suggested including of $1/N$ corrections [17], or bulk gravitational anomaly [18] could induce a nonzero result.

String-net condensation provides a natural physical mechanism for TO in gauge theories [7]. Intuitively, there are two necessary ingredients for string-net condensation: deconfined gauge degrees of freedom and global gauge-invariant constraint [19]. A simple system which satisfies these two conditions is the high-temperature deconfined phase of a weakly-coupled $U(N)$ gauge theory on a compact manifold [20,21]. For simplicity, we consider the free theory on $\mathbb{S}^3$. The question now becomes, how is the nontrivial topological property in such a thermal phase related to a nontrivial TEE defined at zero temperature? There are two different ways to think about this. First, consider the entanglement entropy of a $4+1$ D bulk gauge theory with the sphere $\mathbb{S}^3$ as the entangling surface. The partition induces non gauge-invariant degrees of freedom on the surface. Previous studies show that it is just the global Gauss's law on the entangling surface that gives rise to the topological term [6,22]. Secondly, the high-temperature limit of the thermal phase defines naturally a dimensionally reduced theory on $\mathbb{S}^3$. TEE in this theory will then be given by the logarithm of the vacuum partition function, which is also well studied in the case of $2+1D$ Chern–Simons gauge theories [15,23]. In this paper we will try to make the relation clear from both points of view, and show that all of them are consistent. In comparison, we also perform the calculation in the confining phase.

The main results in the deconfined phase will be derived in the next section. In section 3 we apply the same procedure to the confining phase. Some discussions are given in the last section.

[1] A pedagogic introduction can be found in Part III of [1]. Full classifications, including many recent progresses, of $2+1D$ bosonic/fermionic TO's are given in [3,4].