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On Higgs self-coupling parameter λ of the standard model

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Abstract

Thinking about the Higgs field doublet H^i of the standard model as obeying, in addition to the usual properties, a Weyl-type equation on the Goldstone 3-sphere of the $U_Y(1) \times SU_L(2)$ gauge symmetry, we study underlying features of the free parameters μ^2 and λ of the classical Higgs potential. We show amongst others that λ may be related to the gauge coupling constants g_Y and g_{su_2} like $\lambda = \frac{g_Y^2}{2\xi'^2} + \frac{g_{su_2}^2}{2\xi^2}$ where ξ and ξ' are real numbers with order of magnitude discussed in paper. We show as well that at electroweak scale the mass of neutral Higgs may be estimated as $M_H \simeq M_Z\sqrt{2} \sim 128.7$ GeV.

1. Introduction

In the Standard Model (SM) of electroweak interactions, the breaking of the $SU_L(2) \times U_Y(1)$ gauge symmetry down to $U_{em}(1)$ is achieved by Higgs mechanism [1–3]. In this method, the complex $SU_L(2)$ Higgs field doublet $H^i = (H^+, H^-)$, carrying as well a unit hypercharge $Y_\Phi = 1$, has a classical potential energy density

$$\mathcal{V}_{higgs} = \mu^2(\bar{H}.H) + \lambda(\bar{H}.H)^2 \tag{1.1}$$

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involving two free parameters: (i) a massive constant $\mu^2 < 0$ and (ii) a dimensionless self-Higgs-coupling constant $\lambda > 0$ allowing a Higgs ground state with non-zero vacuum expectation value $\upsilon = \sqrt{\frac{-\mu^2}{2\lambda}} \simeq 246$ GeV. Through the Higgs doublet couplings to the other SM fields; the Higgs VEV gives masses to fermionic particles as well as to the $W^{\pm}$ and Z^0 gauge bosons; the $W^{\pm}$-mass M_W and the neutral Higgs mass M_H are respectively related to the VEV as $\frac{g_{su_2}}{2}\upsilon$ and $\upsilon\sqrt{2\lambda}$. Clearly, the real μ and λ, together with gauge and Yukawa couplings, are basic quantities; their knowledge and the conditions they should obey for vacuum stability up to very high field values are therefore of major importance for SM and beyond; for reviews on some of these aspects see for example [4–8] and references therein.

Motivated by LHC experiments on observation of a Higgs like particle at ∼126 GeV [9,10], and extending ideas dealing with effects induced by external fields and spin–orbit coupling in condensed matter systems [11,12], we focus in this paper on the underlying properties of the parameters μ^2 and λ of the classical Higgs potential $\mathcal{V}_{higgs}$ by using a new method of approaching them. Thinking about the quartic term of $\mathcal{V}_{higgs}$ as resulting from an underlying interaction manifested at the electroweak energy scale as a kind of *isocurrent–isocurrent* couplings[1] like

$$\lambda(\bar{H}.H)^2 = \lambda_1(\bar{H}\vec{\tau}H)^2 + \lambda_2(H\vec{\tau}H).(\bar{H}\vec{\tau}\bar{H}) \tag{1.2}$$

we find that the $\mathcal{V}_{higgs}$ given by (1.1) may be interpreted as an effective Lagrangian density $\mathcal{L}_{eff}$ of the Higgs field H^i on a 3-sphere $\mathbb{S}^3$ fibered on space time $\mathbb{R}^{1,3}$; that is a Higgs field doublet living in $\mathbb{R}^{1,3} \times \mathbb{S}^3$. We find as well that, when restricting the analysis of the Higgs doublet to the fiber $\mathbb{S}^3$ with local coordinates (ψ, θ, φ), the quadratic mass term in the Higgs doublet reads as $(c_{--}H^-H^- + hc)$ or explicitly like $\mu^2\varrho^2[\sin^2\frac{\theta}{2}\cos(\psi - \varphi)]$ with ϱ standing for the neutral Higgs that we observe in space time. This quadratic term in ϱ reduces precisely to the first term of right-hand side of Eq. (1.1) when setting $\psi = \varphi$ and $\theta = \pi$; two relations obtained by solving stability conditions of the Higgs doublet on $\mathbb{S}^3$ fiber. We find, moreover, that the Higgs self-coupling constant λ can be related to the gauge coupling constants as $\lambda = \frac{g_Y^2}{2\xi'^2} + \frac{g_{su_2}^2}{2\xi^2}$ where ξ'^2 and ξ^2 are two numbers whose physical interpretation and their classical estimation at electroweak and Planck scales will be discussed in Sections 5 and 6 of this paper. The relationship $\lambda = \lambda(g, g')$ suggests, in particular, the existence of a link between the Higgs M_H and the gauge bosons M_W and M_Z masses like the relation $M_H \simeq M_Z\sqrt{2}$ to be derived in present study. This link between M_H and M_Z should not be viewed as a strange thing since typical connections like $4M_H^2 \simeq 2m_T^2 \simeq \upsilon^2$ relating M_H to the top-quark mass and the Higgs VEV are experimental facts of SM [8,13].

The presentation of the paper is as follows: In Section 2, we introduce useful aspects of the Higgs doublet H^i on 3-sphere as well as in the $\mathbb{R}^{1,3} \times \mathbb{S}^3$ fibration. In Section 3, we derive the Weyl-type equation satisfied by H^i living on $\mathbb{S}^3$ fiber; it is a particular field equation interpreted as a free field equation on $\mathbb{S}^3$; its generalization including mass and interaction terms is analyzed in Section 4. In Section 5, we show how to extend the $SU_L(2) \times U_Y(1)$ gauge symmetry to the $\mathbb{S}^3$-fiber and derive the effective potential modeling Higgs interactions on $\mathbb{S}^3$. Then we work out the link with the usual potential $\mathcal{V}_{higgs}$ of the standard model viewed in the space time. In Section 6, we give a conclusion and make comments. In Appendices A and B, we collect some technical details on the hyper- and the isospin-vector fields living on the 3-sphere of the fibration. In Appendix C, we recall recent results on RG flows of running couplings in SM.

[1] The $H\vec{\tau}H$ and $\bar{H}\vec{\tau}\bar{H}$ have two units of hypercharges contrary to the hermitian $\bar{H}\vec{\tau}H$.